Welfare Consequences of Sustainable Finance*

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Abstract

We model the welfare consequences of portfolio mandates that restrict investors to hold firms with net-zero carbon emissions. To qualify for these mandates, value-maximizing firms have to accumulate decarbonization capital. Qualification lowers a firm's required rate of return by its decarbonization investments divided by Tobin's q, i.e., the dividend yield shareholders forgo to address the global-warming externality. The welfare-maximizing mandate approximates the first-best solution, yielding welfare gains compared to laissez faire by mitigating the weather disaster risks resulting from carbon emissions. Our model generates transitions to steady-state decarbonization-to-productive capital ratios that we use to evaluate the optimality of proposed net-zero targets.

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1 Introduction

Sustainable finance mandates, whereby asset portfolios are restricted to firms that can meet net-zero emissions targets, are increasingly embraced by the financial sector. Prominent mandates include the Glasgow Financial Alliance for Net Zero, which has commitments from 450 financial firms across 45 countries with $130 trillion of assets (*Wall Street Journal* on November 7, 2021), and the Network for Greening the Financial System (NGFS), which supports net-zero pledges by central banks.

These mandates are meant to address the global-warming externality by influencing the firms' costs of capital, thereby incentivizing them to reform. Following Intergovernmental Panel on Climate Change (IPCC) mitigation pathways (Rogelj et al., 2018), major corporations have announced audited plans to meet net-zero emissions targets by accumulating decarbonization capital, including renewables, afforestation and reforestation, soil carbon sequestration, bioenergy with carbon capture and storage (BECCs), and direct air capture (DAC).[1]

While prior work on socially responsible investing indicates that divestment and the cost-of-capital channel can be a material incentive for firms to reform,[2] there remains challenging questions regarding the welfare consequences of these mandates. First, how close to the first-best outcomes can these mandates get us when it comes to mitigating global warming? Put another way, can mandates be a viable tool to address the global-warming externality when there are imminent risks of a climate tipping point—an absorbing state characterized by more frequent weather disasters (Lenton et al., 2008; Collins et al., 2019; National Academy of Sciences, 2016) that significantly increases the social cost of carbon (Cai et al., 2015;

[1]The European Union and likely the Security Exchange Commission are addressing greenwashing by requiring investors to disclose the carbon emissions of firms in their portfolios.

[2]The first model analyzing the impact of green mandates on the required rate of return is cast in a static CARA setting (Heinkel, Kraus, and Zechner, 2001). Hong and Kacperczyk (2009) show how ethical investing mandates affect the sin companies' costs of capital. Recent work, e.g., Pastor, Stambaugh, and Taylor (2019) and Pedersen, Fitzgibbons, and Pomorski (2020), model how non-pecuniary tastes of green investors influence cross-sectional asset prices in a CAPM setting or in a setting with financial constraints (Oehmke and Opp, 2020). While exits or screens are the predominant forms of mandates, mandates need not only be passive but also active via voting for environmentally friendly policies (Gollier and Pouget, 2014; Broccardo, Hart, and Zingales, 2020).

Cai and Lontzek, 2019)? Second, how should the corporate sector optimally decarbonize given trade-offs between the costs of accumulating decarbonization capital and the benefits of averting catastrophic consequences of global warming for the society as a whole?

To address these issues, we introduce decarbonization capital into a dynamic stochastic general-equilibrium model with the standard capital stock, which serves as both the input for producing a homogeneous good and also the source of carbon emissions (Nordhaus, 2017; Jensen and Traeger, 2014). Decarbonization capital only offsets carbon emissions, has no productive role, comes at the expense of forgone corporate investments or dividend payouts, and also faces capital adjustment costs. More decarbonization relative to productive capital delays a climate tipping point, which is modeled as a Poisson jump process from a "Good" climate state with infrequent weather disasters to an absorbing "Bad" climate state with frequent weather disasters (Lontzek et al., 2015).

Weather disasters in both "Good" and "Bad" climate states, also modelled as jump processes with time-varying arrival rates, destroy both productive and decarbonization capital stocks and lead to significant welfare losses for households with Epstein-Zin recursive utility (Rietz, 1988; Barro, 2006; Pindyck and Wang, 2013; Martin and Pindyck, 2015).[3] To effectively manage climate tipping-point risk, more decarbonization capital stock, which also mitigates weather disasters in both climate states, is needed for an economy with a larger productive capital stock.

Since the decarbonization costs are borne by the firm but its benefits are enjoyed by society in the form of a lower aggregate risk, there is an externality in the economy that can be addressed by sustainable finance mandates. A mandate is composed of the fraction of aggregate wealth that is restricted for sustainable investment and a qualification standard for each firm choosing to be sustainable. In equilibrium, a sufficiently large fraction of *ex ante* identical firms choose to meet the qualification standard so that they are included in the representative investor's sustainable-firm portfolio.

[3]Models with time-varying disaster arrival rates (Gabaix, 2012; Gourio, 2012; Collin-Dufresne, Johannes, and Lochstoer, 2016; Wachter, 2013) have been shown to be quantitively important to simultaneously explain business cycles and asset price fluctuations.

Our analysis has three main sets of results. First, the required rate of return for a sustainable firm is lower than that for an unsustainable firm. The wedge between the two types equals the required mitigation spending (to fund the aggregate decarbonization capital accumulation) for a sustainable firm divided by its Tobin's q, i.e., the dividend yield a sustainable firm's shareholders forgo to address the global-warming externality or greenium. This required rate of return formula is due to the equilibrium result that Tobin's q, for sustainable and unsustainable firms must be the same so that firms are indifferent between being sustainable or not.[4] Additionally, sustainable and unsustainable firms invest and hence grow at the same rate (path by path) over time. This is because 1) investment is connected to Tobin's q via first-order conditions for both types of firms (Hayashi, 1982) and 2) both types of firms have the same Tobin's q. Finally, sustainable firms must lower their payouts to their shareholders in order to fund their mitigation spendings in order to enjoy lower costs of capital and keep the growth of all firms the same.

The premium for sustainable stocks (i.e. greenium) in our model arises for a reason different from the standard mechanism in the literature, e.g., Heinkel, Kraus, and Zechner (2001), Hong and Kacperczyk (2009), Pastor, Stambaugh and Taylor (2019). In these papers, a group of investors who are financially unconstrained have to be indifferent between investing in sustainable firms or not in equilibrium at the margin. Mandates force these unconstrained investors to take concentrated positions in unsustainable firms. To the extent stocks are imperfect substitutes, e.g., due to idiosyncratic shocks, unconstrained investors will demand a higher required rate of return for unsustainable firms. In contrast, in our model, there is no idiosyncratic risk and portfolio shares are fixed. The reason the mandate has an effect in our model is that value-maximizing firms have to be indifferent between being sustainable or not.

Second, we compare households' welfare in a competitive-markets economy augmented with welfare-maximizing mandates with that in the first-best economy. Whereas the plan-

[4] The decarbonization capital, which is unproductive and does not contribute to output, sits in the firm's assets but is not priced by markets other than through the mandate qualification mechanism.

ner jointly chooses mitigation and productive investments, firms in the market economy with welfare-maximizing mandates choose productive investments taking as given the mitigation spending required by the welfare-maximizing mandate path. The welfare-maximizing mandate in general depends on the climate state, the productive capital stock and the ratio of decarbonization-to-productive capital. Given a sufficiently large fraction of aggregate wealth that is restricted to mandates, we solve for the optimal required firm mitigation spending that maximizes welfare in the market economy. There tends to be too much investment and too little consumption in the welfare-maximizing mandated market economy compared to the first-best economy.

We prove that incorporating another policy instrument, e.g., an investment tax, into the market economy with optimal mandates (discussed above) can attain the first-best. Quantitatively, we show introducing the welfare-maximizing mandate alone into the market economy well approximates the first-best outcomes. In other words, the optimal mandate can be a useful tool to address the global warming externality.

Third, our model generates transitions to steady-state decarbonization-to-productive capital ratios that can be used to evaluate the optimality net-zero targets proposed by policymakers. When the adjustment costs of productive and decarbonization capital are close, the optimal path in the mandated market economy implies a rapid transition to a high steady-state ratio of decarbonization-to-productive capital stock, much in the way that policy makers are hoping with 2030 or 2050 net-zero targets in the Paris agreement.

Even though decarbonization capital is entirely unproductive, its disaster-risk mitigation benefits are such that as the mandated market economy decarbonizes, the aggregate risk of economic growth is reduced. Hence, there are increases in investment, growth, and household welfare over time as the economy reaches steady state. Asset prices, including the stock-market risk premium and the aggregate Tobin's q, also respond favorably to the lower aggregate risk resulting from the accumulation of decarbonization capital. Our model offers a rationale for positive growth over the net-zero transition that do no require assumptions that renewables are highly cost effective (see, e.g., European Union (2022) projections). But even modestly higher adjustment costs for accumulating decarbonization capital result in a dramatically slower transition to a much lower steady-state decarbonization-to-capital ratio.

Our paper differs from the two sector model of Eberly and Wang (2009), where investors' preferences for portfolio diversification is the key force. Our paper builds on Hong, Wang and Yang (2022), who model the regional-level mitigation of weather disasters, and the optimal capital tax to stimulate the first-best level of flow spending for preparedness. Our paper contributes to the emerging climate-finance literature on the role of the financial system in addressing global warming (see Hong, Karolyi, and Scheinkman (2020) for an overview). Bansal, Ochoa, and Kiku (2017) use a long-run risk model to evaluate the impact of higher temperature on growth stocks. Barnett, Brock, and Hansen (2020) provide an asset-pricing framework to confront climate model uncertainty. Engle et al. (2020) develop a method to hedge climate risks through trading of stock portfolios. Piazessi, Papoutsi, and Schneider (2022) develop a deterministic multisector growth model with climate externalities and financial frictions to study the environmental impact of unconventional monetary policy.

2 Model

2.1 Climate State

Consider the following climate-transition model. Let $\mathcal{S}_t$ denote the climate state at time t. The economy starts from the good climate state ($\mathcal{G}$) and stochastically transitions to the bad state ($\mathcal{B}$) at a stochastic rate of $\zeta_t > 0$. Moreover, we assume that this climate transition is permanent in that the $\mathcal{B}$ state is absorbing. In both climate states, there are also weather disaster shocks, e.g., hurricanes and wildfires that destroy capital. But the good climate state ($\mathcal{G}$) has less frequent weather disasters than the bad state ($\mathcal{B}$). We model these weather disaster shocks and the climate state transition via jumps to be discussed in detail later. Next, we introduce the production side of the economy.

2.2 Firm Production and Productive Capital (K) Accumulation

There is a continuum of firms endowed with the same production function and capital accumulation technology. In both climate states ($\mathcal{G}$ and $\mathcal{B}$), each firm's output at time t, Y_t, is proportional to its contemporaneous productive capital stock, K_t:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity. This is a version of widely-used AK models in macroeconomics and finance. This simplifying assumption makes our model tractable and allows us to focus on the impact of the financial investment mandate on equilibrium asset pricing and resource allocation.[5]

2.2.1 Investment

Let I_t denote a firm's investment. Also in both $\mathcal{B}$ and $\mathcal{G}$ climate states, the firm's productive capital stock, K_t, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{W}_t - (1 - Z)K_{t-} d\mathcal{J}_t \ . \tag{2}$$

As in Lucas and Prescott (1971), Hayashi (1982), and Jermann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K, and thus

$$\Phi(I, K) = \phi(i)K \ , \tag{3}$$

where $i = I/K$ is the investment-capital ratio and $\phi(\,\cdot\,)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. The installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital exceeds one. The second term captures continuous (Brownian motion) shock to capital $\{\mathcal{W}_t\}$ (common to all firms) and the parameter σ is the diffusion volatility. Next, we introduce disaster shocks.

[5] To ease exposition, we assume that all firms start with the same initial capital stock level, K_0, although our model can be generalized to allow for heterogeneous levels of initial K_0. We could also generalize our model by introducing idiosyncratic shocks across firms. Our aggregation results would remain valid as long as firms can also perfectly hedge idiosyncratic shocks at no cost.

2.2.2 Weather Disaster (Jump) Shocks

In both climate states ($\mathcal{G}$ and $\mathcal{B}$), the firm's capital stock K is subject to an aggregate jump shock due to weather disasters. We capture weather disaster shocks via the third term in (2), where $\{\mathcal{J}_t\}$ is a (pure) jump process driving weather disaster arrivals with a climate-state-dependent arrival rate $\{\lambda_t^{\mathcal{S}_t}\}$ process.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the firm's capital stock K_{t-}, as $Z \in (0, 1)$ is the recovery fraction. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = 0.85$.) There is no limit to the number of these weather disaster shocks. If a jump does not arrive in state $\mathcal{S}_t$, i.e., $d\mathcal{J}_t = 0$, the third term disappears. Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) of the recovery fraction, Z, conditional on a jump arrival, respectively. We assume that the cdf $\Xi(Z)$ and pdf $\xi(Z)$ are time invariant.

In a given climate state $\mathcal{S}_t$ ($\mathcal{B}$ or $\mathcal{G}$) at time t, we model the stochastic damage upon the arrival of a weather disaster by assuming that the recovery fraction, $Z \in (0, 1)$, of capital stock is governed by the following cdf (Barro and Jin, 2011; Pindyck and Wang, 2013):

$$\Xi(Z) = Z^{\beta} , \tag{4}$$

where $\beta > 0$ is a constant. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta > \max\{\gamma - 1, 0\}$. That is, the damage caused by a weather disaster arrival follows a fat-tailed power-law function (Gabaix, 2009).

2.2.3 Firm Investment, Dividends, and Mitigation Spending (Contribution)

At any time t, the firm uses its output AK_t to finance investment I_t, pay cash flows (dividends) CF_t to shareholders, and make mitigation spending X_t contributing towards the aggregate decarbonization capital accumulation to be described in detail soon. Therefore,

$$Y_t = AK_t = I_t + CF_t + X_t \, . \tag{5}$$

We use **boldfaced** notations for aggregate variables. Next, we introduce emissions, emission removals, and the dynamics of the aggregate decarbonization capital stock $\mathbf{N}$.

2.3 Aggregate Emissions, Emission Removals, and Decarbonization Capital Stock N

We assume that aggregate emissions $\mathbf{E}_t$ is proportional to the aggregate productive capital stock $\mathbf{K}_t$:

$$\mathbf{E}_t = \mathbf{e}\mathbf{K}_t \,, \tag{6}$$

where $\mathbf{e} > 0$ is a constant. The aggregate capital stock $\mathbf{K}_t$ and emissions $\mathbf{E}_t$ equal the sum (integral) of each firm's capital stock K_t and emissions E_t: $\mathbf{K}_t = \int K_t^\nu d\nu$ and $\mathbf{E}_t = \int E_t^\nu d\nu$, respectively.[6] That is, aggregate emissions increase linearly with the size of the production sector of the economy, which is measured by the aggregate capital stock $\mathbf{K}$ or equivalently GDP ($A\mathbf{K}$). Similarly, we assume that the aggregate emission removals $\mathbf{R}_t$ is proportional to the aggregate decarbonization capital stock $\mathbf{N}_t$:

$$\mathbf{R}_t = \varrho \mathbf{N}_t \,, \tag{7}$$

where $\varrho > 0$ is a constant. Both aggregate emissions $\mathbf{E}_t$ and carbon removals $\mathbf{R}_t$ are given by an "AK"-type of technology, as we can see from (6) and (7).

Let $\mathbf{X}_t$ denote the aggregate mitigation spending (investment), which equals the sum of mitigation spending contributions by all firms: $\mathbf{X}_t = \int X_t^\nu d\nu$. The aggregate decarbonization capital stock $\mathbf{N}$ evolves as follows:

$$\frac{d\mathbf{N}_t}{\mathbf{N}_{\mathbf{t}-}} \;=\; \omega(\mathbf{X}_{t-}/\mathbf{N}_{\mathbf{t}-})dt + \sigma d\mathcal{W}_t - (1-Z)d\mathcal{J}_t \,. \tag{8}$$

The control $\mathbf{X}_{t-}/\mathbf{N}_{\mathbf{t}-}$ in (8) for $\mathbf{N}$ accumulation at the aggregate level is analogous to the investment-capital ratio I_{t-}/K_{t-} in (2) for productive capital (K) accumulation at the firm level. That is, absent jumps, $\omega(\mathbf{X}_{t-}/\mathbf{N}_{\mathbf{t}-})$, the drift of $d\mathbf{N}_t/\mathbf{N}_{t-}$, is analogous to $\phi(I_{t-}/K_{t-})$, the drift of dK_t/K_{t-}. We assume that $\omega(\,\cdot\,)$ is increasing and concave as we do for $\phi(\,\cdot\,)$. This specification captures the idea that changing $\mathbf{N}$ rapidly is more costly than changing it

[6] We integrate over a continuum of firms with respect to the measure ν. See Sun (2006) for technical conditions under which we can construct the associated probability and agent measures that allow invoking a law of large numbers.

slowly. As we show later, the adjustment costs for $\mathbf{N}_t$ has first-order implications on welfare implications and the transition path towards the net-zero target.[7]

Equation (8) implies that the growth rate for the decarbonization capital stock $\mathbf{N}$, $d\mathbf{N}_t/\mathbf{N}_{t-}$, is subject to the same diffusion and jump shocks as the growth rate of the aggregate productive capital stock $\mathbf{K}$, $d\mathbf{K}_t/\mathbf{K}_{t-}$. Recall that the productive capital stock at the aggregate level follows the same process as at the firm level: $d\mathbf{K}_t/\mathbf{K}_{t-} = dK_t/K_{t-}$ path by path, e.g., for each realized jump and recovery fraction Z.

Let $\mathbf{n}_t$ denote the aggregate decarbonization-productive capital ratio:

$$\mathbf{n}_t = \frac{\mathbf{N}_t}{\mathbf{K}_t}\,. \tag{9}$$

Using Ito's lemma, we obtain the following dynamics for $\mathbf{n}_t$:

$$\frac{d\mathbf{n}_t}{\mathbf{n}_{t-}} = \left[\omega(\mathbf{x}_{t-}/\mathbf{n}_{t-}) - \phi(\mathbf{i}_{t-})\right] dt\,. \tag{10}$$

Note that there is no uncertainty for the dynamics of $\mathbf{n}_t$ in our model. This is because productive and decarbonization capital stocks are subject to the same jump-diffusion growth shocks.[8] We next introduce the climate tipping-point and weather disaster arrival rates.

2.4 Tipping-point Arrival and Weather Disaster Arrival Rates

Let $\widetilde{\mathcal{J}}_t$ denote the climate tipping-point arrival process. Conditional on being in the good climate state at time t, $\mathcal{S}_t = \mathcal{G}$, global warming increases the arrival rate of the climate tipping point. As state $\mathcal{B}$ is assumed to be absorbing, there are no further climate-state transitions once the economy is in state $\mathcal{B}$. (For notional convenience, we will sometimes denote the arrival rate of the climate tipping point by $\zeta_t^{\mathcal{S}_t}$ with the understanding that $\zeta_t^{\mathcal{G}} > 0$ and $\zeta_t^{\mathcal{B}} = 0$.)

First, we assume that the tipping-point arrival rate $\zeta_t^{\mathcal{G}}$ is increasing in the aggregate emissions $\mathbf{E}_t$ and decreasing in the aggregate emissions removals $\mathbf{R}_t$. Similarly, we assume

[7] In our model, whether firms do mitigation spending on their own (e.g., planting trees by themselves) or contribute resources to the planner who plants trees on behalf of all firms, the solution is the same. This is because a firm's mitigation spending yields only public benefits and no firm-specific benefit. We choose to specify an aggregate decarbonization capital accumulation process throughout our paper.

[8] Note that $\mathbf{n}_t$ is continuous even when the climate state transitions from $\mathcal{G}$ to $\mathcal{B}$.

that the weather disaster arrival rates in both climate states, $\lambda_t^{\mathcal{G}}$, and $\lambda_t^{\mathcal{B}}$, are also increasing in the aggregate emissions $\mathbf{E}_t$ and decreasing in the aggregate emissions removals $\mathbf{R}_t$. As $\mathbf{E}_t = \mathbf{e}\mathbf{K}_t$ and $\mathbf{R}_t = \varrho\mathbf{N}_t$ (see equations (6) and (7)), we may write the three transition rates, ($\zeta_t^{\mathcal{G}}$, $\lambda_t^{\mathcal{G}}$, and $\lambda_t^{\mathcal{B}}$), as functions that are increasing in $\mathbf{K}_t$ and decreasing in $\mathbf{N}_t$.

We assume that the effects of $\mathbf{K}_t$ and $\mathbf{N}_t$ on the three transition rates ($\zeta_t^{\mathcal{G}}$, $\lambda_t^{\mathcal{G}}$, and $\lambda_t^{\mathcal{B}}$) can be summarized via $\mathbf{n}_t$. That is, $\zeta_t^{\mathcal{G}}$, $\lambda_t^{\mathcal{G}}$, and $\lambda_t^{\mathcal{B}}$ are all homogeneous of degree zero in $\mathbf{K}_t$ and $\mathbf{N}_t$. We thus write these rates as functions of the scaled aggregate decarbonization stock $\mathbf{n}_t = \mathbf{N}_t/\mathbf{K}_t$: $\zeta_t^{\mathcal{G}} = \zeta(\mathbf{n}_t;\mathcal{G})$, $\lambda_t^{\mathcal{G}} = \lambda(\mathbf{n}_t;\mathcal{G})$, and $\lambda_t^{\mathcal{B}} = \lambda(\mathbf{n}_t;\mathcal{B})$.[9] Recall $\zeta_t^{\mathcal{B}} = \zeta(\mathbf{n}_t;\mathcal{B}) = 0$.

As decarbonization probabilistically delays the tipping point and reduces the weather-disaster arrival rates, we assume $\zeta'(\mathbf{n}_t;\mathcal{G}) < 0$, $\lambda'(\mathbf{n}_t;\mathcal{G}) < 0$, and $\lambda'(\mathbf{n}_t;\mathcal{B}) < 0$. Additionally, we assume that the marginal benefits (e.g., decreasing the climate tipping-point arrival rate and reducing the frequencies of weather disaster shocks) of accumulating decarbonization capital stock decreases as $\mathbf{n}_t$ increases: $\zeta''(\mathbf{n}_t;\mathcal{G}) > 0$, $\lambda''(\mathbf{n}_t;\mathcal{G}) > 0$, and $\lambda''(\mathbf{n}_t;\mathcal{B}) > 0$. That is, the absolute value for the derivative of the climate tipping-point arrival rate, $|\zeta'(\mathbf{n}_t)|$, decreases with $\mathbf{n}_t$. Similarly, the marginal effect (magnitude wise) of $\mathbf{N}$ on the change of $\lambda^{\mathcal{S}_t}$ decreases as $\mathbf{N}$ increases in that $\lambda''(\mathbf{n}_t;\mathcal{S}_t) > 0$.

Finally, to capture the idea that weather disasters are more frequent in the $\mathcal{B}$ state than in the $\mathcal{G}$ state conditional on $\mathbf{n}_t$, we assume $\lambda_t(\mathbf{n}_t;\mathcal{G}) < \lambda_t(\mathbf{n}_t;\mathcal{B})$ for all $\mathbf{n}_t$. We specify the functional forms for $\lambda_t(\mathbf{n}_t;\mathcal{G})$, $\lambda_t(\mathbf{n}_t;\mathcal{B})$, and $\zeta(\mathbf{n}_t;\mathcal{G})$ in Section 6.

As climate transition and weather disaster shocks are aggregate, how much each individual firm spends on mitigation has no impact on its own payoff. Therefore, absent mandates or other incentive programs, firms have no incentives to mitigate on their own in a competitive market economy.

2.5 Sustainable Investment Mandate

The sustainable investment mandate requires the representative agent to invest a constant fraction ($\alpha > 0$) of the entire portfolio (aggregate wealth) in sustainable firms, referred to as

[9]To make the dependence of $\zeta_t^{\mathcal{S}_t}$ and $\lambda_t^{\mathcal{S}_t}$ on $\mathbf{n}_t$ and $\mathcal{S}_t$ explicit, we write $\zeta_t^{\mathcal{S}_t} = \zeta(\mathbf{n}_t;\mathcal{S}_t)$ and $\lambda_t^{\mathcal{S}_t} = \lambda(\mathbf{n}_t;\mathcal{S}_t)$. This homogeneity assumption is consistent with sustainable long-term balanced growth.

type-S firms, at all time t when allocating assets.

On the supply side, a portfolio of S firms and a portfolio of U firms will arise endogenously in equilibrium, which we refer to as the S portfolio and U portfolio, respectively. For a firm to qualify to be type-S, it has to spend at least $M_t = m_t K_t$ at all time t. That is, a firm at least spends m_t for each unit of its productive capital K_t on mitigation by contributing to the accumulation of the aggregate decarbonization capital stock, which delays the tipping-point arrival and reduces the weather disaster shock arrival rates. A firm is then qualified to be included in the S-portfolio, if and only if its mitigation spending X_t satisfies:

$$X_t \geq M_t \,. \tag{11}$$

Otherwise, it is a type-U unsustainable firm.

The S and U portfolios include all the S and U firms, respectively. Let $\mathbf{Q}_t^S$ and $\mathbf{Q}_t^U$ denote the aggregate market value of the S portfolio and of the U portfolio at t, respectively. The total market capitalization of the economy, $\mathbf{Q}_t$, is given by $\mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U$. In equilibrium, the investment mandate requires that the total capital investment in the S portfolio, $\mathbf{Q}_t^S$, has to be at least an α fraction of the total stock market capitalization $\mathbf{Q}_t$:

$$\mathbf{Q}_t^S \geq \alpha \mathbf{Q}_t \,. \tag{12}$$

Next we turn to the demand side of the economy.

2.6 Dynamic Consumption and Asset Allocation

The representative agent makes consumption, asset allocation, and risk management decisions. We use individual and aggregate variables for the agent interchangeably as we have a continuum of identical agents (with unit measure). For example, the aggregate wealth, $\mathbf{W}_t$, is equal to the representative agent's wealth, W_t, in equilibrium. Similarly, the aggregate consumption, $\mathbf{C}_t$, is equal to the representative agent's consumption, C_t.

The representative agent has the following investment opportunities: (a) the S portfolio which includes all the sustainable firms; (b) the U portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a risk-free interest rate r^f process determined in equilibrium.[10]

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the homothetic recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that we may express the agent's value-function process, $\{V_t; t \geq 0\}$, as follows:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] , \tag{13}$$

where $f(C, V)$ is known as the normalized aggregator given by

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^\chi}{((1-\gamma)V)^{\chi-1}} . \tag{14}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\chi = (1 - \psi^{-1})/(1 - \gamma)$. Unlike expected utility, recursive preferences as defined by (13) and (14) disentangle the coefficient of relative risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\chi-1}$, which depends not only on current consumption but also (through the value function V) on the trajectory of future consumption.

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the follow-up long-run risk literature.[11]

[10] To be precise, as markets are dynamically spanned, the economy also has actuarially fair insurance claims for each weather disaster arrival (with every possible recovery fraction Z) and the insurance contracts contingent on climate transition as well as diffusion shocks. But we suppress these zero-net-supply claims since they do not change the allocations in the economy as shown in Pindyck and Wang (2013) and Hong, Wang, and Yang (2022).

[11] If $\gamma = \psi^{-1}$ so that $\chi = 1$, the recursive utility (13) turns into the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable (normalized) aggregator:

$$f(C, V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho\, V.$$

2.7 Competitive Equilibrium with Mandates

Let $\mathbf{Y}_t$, $\mathbf{C}_t$, $\mathbf{I}_t$, and $\mathbf{X}_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Using an individual firm's resource constraint (5) and adding across all type-S and type-U firms, we obtain the aggregate resource constraint in the economy:

$$\mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \mathbf{X}_t \,. \tag{15}$$

We define the competitive equilibrium subject to the investment mandate defined earlier as follows: (i.) the representative agent dynamically chooses consumption and asset allocation among the S portfolio, the U portfolio, and the risk-free asset subject to the investment mandate; (ii.) each firm chooses its status (S or U) via mitigation spending and investment I to maximize its market value; (iii.) all firms that choose sustainable investment policies are included in the S portfolio and all remaining (unsustainable) firms are included in the U portfolio; and (iv.) all markets clear.

The market-clearing conditions at each time t include (i.) the representative agent's demand for the S portfolio equals the total supply by firms choosing to be sustainable; (ii.) the representative agent's demand for the U portfolio equals the total supply by firms choosing to be unsustainable; (iii.) the net supply of the risk-free asset is zero; and (iv.) the goods market clears, i.e., the aggregate resource constraint given in (15) holds.

2.8 Comments on Model Assumptions

We highlight three key sets of assumptions that are made to gain tractability.

2.8.1 Firm Decarbonization Technology

Following the carbon-externality literature, we assume that it does not matter which firm does the clean-up (Salanie, 2000). In principle, we can allow firms to have different emission intensities. Any firm could be sustainable and spend on the clean-up. For decarbonization technologies such as direct air capture (DAC), this seems a good assumption since these DAC plants could be built by many firms in many industries (e.g., DAC investments made by Microsoft). But there are other technologies where it might be more efficient for firms in certain industries (e.g., dirty industries) to execute. In this instance, the investment mandate would naturally also depend on the types of industries that firms are in and other factors.

2.8.2 Carbon Cycle and Damage Function

We have made simplifying assumptions regarding the carbon cycle and damage functions in our model. Below we discuss these simplifications and some potential generalizations. First, in reality, emissions increase the stock of carbon which affects temperature with a delay. In our model the stock of carbon is assumed to immediately influence the climate tipping-point and weather disaster arrivals. We can generalize our model by allowing for a lag between the time at which an investment in the decarbonization capital $\mathbf{N}$ is made and the time at which this decarbonization investment has a risk-mitigating effect. Introducing this time lag will introduce additional technical complications into our analysis. This is because we need to keep track of both the decarbonization capital that is mitigating the aggregate risk as well as other accumulated decarbonization stock $\mathbf{N}$ that will mitigate in the future.

Second, the climate tipping-point arrival rate ζ_t depends on cumulative emissions in the atmosphere. We assume that this cumulative emissions is well approximated using $\mathbf{n} = \mathbf{N}/\mathbf{K}$ in our model. An alternative specification is that the arrival rate ζ_t depends on $(\mathbf{K}_t - \mathbf{N}_t)$. There is no guidance in climate science on which approximation is more sensible *per se*. How important the functional form assumption is also depends on the underlying decarbonization technologies. But the latter level-based specification is not tractable in our growth stationary economy where economic damages of disasters increase with $\mathbf{K}$.

2.8.3 Exposures of Productive and Decarbonization Capital to Disaster Shocks

Weather disaster shocks in our model are assumed to affect the stochastic growth of productive and decarbonization capital equally. This is a simplification since it makes the dynamics of $\mathbf{n}_t$ deterministic. This property allows us to conveniently analyze the transition dynamics towards net-zero target over time. If we were to allow the stochastic growth of productive and decarbonization capital stocks to respond differently to jump and diffusion shocks, $\mathbf{n}_t$

would be stochastic, but we still have the homogeneity property and hence will not lose much tractability.

3 Equilibrium Solution with Mandates

In this section, we obtain and analyze the equilibrium solution with the sustainable finance mandate. A firm has to spend the minimal required m_t fraction of its productive capital stock K_t to qualify as a sustainable firm at time t. While spending on aggregate risk mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio. We work within the set of m_t specifications where we can write m_t as a function of $\mathbf{n}_t$ and climate state $\mathcal{S}_t$: $m_t = m(\mathbf{n}_t; \mathcal{S}_t)$. We assume that a firm's mitigation is observable and contractible. We first solve for the equilibrium in Subsections 3.1-3.3 for a given m_t process and then solve for the welfare-maximizing m_t in Subsection 3.4. Finally, we comment on our model assumptions and equilibrium in Subsection 3.5.

3.1 Firm Optimization

A value-maximizing firm chooses whether to be sustainable or unsustainable taking the sustainable investment mandate into account. First, we pin down mitigation spending by both types of firms: X_t^U and X_t^S. As mitigation spending has no direct benefit for the firm, if the firm chooses to be U, it will set $X_t^U = 0$ for all t. Moreover, even if a firm chooses to be an S firm, it has no incentive to spend more than M_t, i.e., (11) always binds for a type-S firm. That is, it is optimal for a sustainable firm to set x_t^S as:

$$x_t^S = \frac{X_t^S}{K_t^S} = m(\mathbf{n}_t; \mathcal{S}_t) \,, \tag{16}$$

where $m_t = m(\mathbf{n}_t; \mathcal{S}_t)$ is the minimal threshold level of a firm's M_t/K_t above which it is qualified to be sustainable. By meeting this mandate, the firm lowers its required rate of return.

Firms are indifferent between the two options in equilibrium. To solve for the equilibrium, first, we solve the following problem for a type-j firm:

$$\max_{I^j, X^j} \mathbb{E}\left(\int_0^\infty e^{-\int_0^t r^j(\mathbf{n}_v;\mathcal{S}_v)dv} CF^j(\mathbf{n}_t;\mathcal{S}_t)dt\right) . \tag{17}$$

In equation (17), $r^j(\mathbf{n}_t;\mathcal{S}_t)$ is the expected cum-dividend return for a type-j firm in equilibrium[12] and $CF^j(\mathbf{n}_t;\mathcal{S}_t)$ is type-j firm's cash flow at t given by

$$CF^S(\mathbf{n}_t;\mathcal{S}_t) = AK_t^S - I_t^S(\mathbf{n}_t;\mathcal{S}_t) - X_t^S(\mathbf{n}_t;\mathcal{S}_t) \text{ and } CF^U(\mathbf{n}_t;\mathcal{S}_t) = AK_t^U - I_t^U(\mathbf{n}_t;\mathcal{S}_t) . \tag{18}$$

Since the fraction of total wealth allocated to meet the sustainability investment mandate is $\alpha \in (0,1]$, the scaled aggregate mitigation spending, $\mathbf{x}_t$, is given by

$$\mathbf{x}_t = \frac{\mathbf{X}_t}{\mathbf{K}_t} = \frac{\alpha X_t^S}{K_t^S} = \alpha x_t^S = \alpha m(\mathbf{n}_t;\mathcal{S}_t) . \tag{19}$$

Exploiting our model's homogeneity property, we conjecture and verify that the equilibrium value of a type-j firm, Q_t^j, at time t must satisfy:

$$Q_t^j = q^j(\mathbf{n}_t;\mathcal{S}_t) K_t^j , \tag{20}$$

where $q^j(\mathbf{n}_t;\mathcal{S}_t)$ is Tobin's q for a type j-firm as a function of $\mathbf{n}_t$ and climate state $\mathcal{S}_t$.

Next, we consider the firm's investment problem when it takes the sustainability mandate $\{m_t = m(\mathbf{n}_t;\mathcal{S}_t) : t \geq 0\}$ as given. The following Hamilton-Jacobi-Bellman (HJB) equation characterizes the firm's value function in climate state $\mathcal{S}$:[13]

$$\begin{aligned} r^j(\mathbf{n};\mathcal{S})Q^j(K^j,\mathbf{n};\mathcal{S}) = \max_{I^j} \; & CF^j(\mathbf{n};\mathcal{S}) + \Phi(I^j,K^j)Q_K^j(K^j,\mathbf{n};\mathcal{S}) + \frac{1}{2}(\sigma K^j)^2 Q_{KK}^j(K^j,\mathbf{n};\mathcal{S}) \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right] \mathbf{n} Q_{\mathbf{n}}^j(K^j,\mathbf{n};\mathcal{S}) \\ & + \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[Q^j(ZK^j,\mathbf{n};\mathcal{S}) - Q^j(K^j,\mathbf{n};\mathcal{S})\right] \\ & + \zeta(\mathbf{n};\mathcal{S})(Q^j(K^j,\mathbf{n};\mathcal{S}') - Q^j(K^j,\mathbf{n};\mathcal{S})) , \end{aligned} \tag{21}$$

[12] Additionally, we impose the standard transversality condition for (17).

[13] A type-j firm's objective (17) implies that $\int_0^u e^{-\int_0^t r^j(\mathbf{n}_v;\mathcal{S}_v)dv} CF^j(\mathbf{n}_t;\mathcal{S}_t)dt + e^{-\int_0^u r^j(\mathbf{n}_v;\mathcal{S}_v)dv} Q_u^j$ is a martingale under the physical measure, where $r^j(\mathbf{n};\mathcal{S})$ is the required rate of return that the firm takes as given. The firm also takes the scaled aggregate decarbonization capital stock $\mathbf{n}$, aggregate mitigation spending $\mathbf{x}(\mathbf{n};\mathcal{S})$, and aggregate investment $\mathbf{i}(\mathbf{n};\mathcal{S})$ as given.

where $\mathcal{S} = \{\mathcal{G}, \mathcal{B}\}$ and $\mathcal{S}'$ denotes the other state. For example, if $\mathcal{S} = \mathcal{G}$, then $\mathcal{S}' = \mathcal{B}$.[14] The left side of (21) is the (cum-dividend) expected return $r^j(\mathbf{n};\mathcal{S})$ times the market value of type-j firm. The first term on the right side is the dividend (cash flow) payment. The second and third terms are the capital accumulation and diffusion volatility effects on the expected capital gains. The last two terms capture the effects of weather disaster arrivals and the climate tipping point arrival on the expected capital gains. The conditional expectation $\mathbb{E}[\,\cdot\,]$ in (21) operates with respect to the distribution of recovery fraction Z and $CF^j(\mathbf{n};\mathcal{S})$ is the cash flow for a type-j firm in climate state $\mathcal{S}$ given by (18). Let $cf^j(\mathbf{n};\mathcal{S}) = CF^j(\mathbf{n};\mathcal{S})/K^j$ denote the scaled cash flow for a type-j firm.

By using our model's homogeneity property, $Q_t^j = q^j(\mathbf{n}_t;\mathcal{S}_t)K_t^j$ for $\mathcal{S} = \{\mathcal{G}, \mathcal{B}\}$, we obtain the following ODE for $q^j(\mathbf{n};\mathcal{S})$, the Tobin's q in the climate state $\mathcal{S}$:

$$\begin{aligned} r^j(\mathbf{n};\mathcal{S})q^j(\mathbf{n};\mathcal{S}) = \max_{i^j} \; & cf^j(\mathbf{n};\mathcal{S}) + (\phi(i^j) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)))q^j(\mathbf{n};\mathcal{S}) \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}q^j_{\mathbf{n}}(\mathbf{n};\mathcal{S}) + \zeta(\mathbf{n};\mathcal{S})(q^j(\mathbf{n};\mathcal{S}') - q^j(\mathbf{n};\mathcal{S}))\,. \end{aligned} \tag{22}$$

The investment FOCs for both S and U firms implied by (22) in both $\mathcal{G}$ and $\mathcal{B}$ states are the following well known conditions in the q-theory literature:

$$q^j(\mathbf{n};\mathcal{S}) = \frac{1}{\phi'(i^j(\mathbf{n};\mathcal{S}))}\,. \tag{23}$$

A type-j firm's marginal benefit of investing equals its marginal q, $q^j(\mathbf{n};\mathcal{S})$, multiplied by $\phi'(i^j(\mathbf{n};\mathcal{S}))$. The investment FOC (23) states that this marginal benefit, $q^j(\mathbf{n};\mathcal{S})\phi'(i^j(\mathbf{n};\mathcal{S}))$, equals one, the marginal cost of investing. The homogeneity property implies that a firm's marginal q is equal to its average q (Hayashi, 1982).

Let $g^j(\mathbf{n};\mathcal{S})$ denote a type-j firm's expected growth rate including the effect of jumps. In state $\mathcal{S}$, the expected growth rate is

$$g^j(\mathbf{n};\mathcal{S}) = \phi(i^j(\mathbf{n};\mathcal{S})) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \zeta(\mathbf{n};\mathcal{S})\frac{q^j(\mathbf{n};\mathcal{S}) - q^j(\mathbf{n};\mathcal{S}')}{q^j(\mathbf{n};\mathcal{S})}\,. \tag{24}$$

The first term captures the investment effect, the second term describes the weather disaster effect, and the last term gives the effect of the climate tipping-point arrival on growth.

[14] Recall that $\zeta(\mathbf{n};\mathcal{B}) = 0$.

As $x^S(\mathbf{n};\mathcal{S}) = m(\mathbf{n};\mathcal{S})$ and $x^U(\mathbf{n};\mathcal{S}) = 0$, we have $cf^S(\mathbf{n};\mathcal{S}) = A - i^S(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S})$ for a type-S firm and $cf^U(\mathbf{n};\mathcal{S}) = A - i^U(\mathbf{n};\mathcal{S})$ for a type-U firm.

3.2 Representative Agent's Optimization

To solve the portfolio-allocation problem, we first introduce the investment opportunities.

3.2.1 Return Dynamics of S and U Portfolios

Let $\mathbf{Q}_t^j$ denote the market value of the type-j portfolio, which includes all type-j firms, where $j = \{S, U\}$. Let $\mathbf{D}_t^j$ and $\mathbf{D}_t^U$ denote the dividends of the type-j portfolio. We later show that the equilibrium cum-dividend return for the type-j portfolio in state $\mathcal{S}$ is:

$$\begin{aligned}\frac{d\mathbf{Q}_t^j + \mathbf{D}_{t-}^j dt}{\mathbf{Q}_{t-}^j} &= r^j(\mathbf{n}_{t-};\mathcal{S})dt + \sigma d\mathcal{W}_t - (1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{S})dt\right) \\ &\quad + \frac{\mathbf{q}^j(\mathbf{n}_{t-}^j;\mathcal{S}') - \mathbf{q}^j(\mathbf{n}_{t-};\mathcal{S})}{\mathbf{q}^j(\mathbf{n}_{t-};\mathcal{S})}\left(d\widetilde{\mathcal{J}}_t - \zeta(\mathbf{n}_{t-};\mathcal{S})dt\right). \qquad (25)\end{aligned}$$

The diffusion volatility equals σ as in (2). The third term on the right side of (25) captures the effect of disasters on return dynamics. The fourth (last) term describes the effect of climate transition from the $\mathcal{G}$ state to the absorbing $\mathcal{B}$ state.[15] Upon the arrival of the tipping point ($d\widetilde{\mathcal{J}}_t = 1$), the percentage change of the portfolio value equals the percentage change of Tobin's q caused by the climate state transition. This is because unlike the weather disaster shock $d\mathcal{J}_t$, the climate state transition shock $\widetilde{\mathcal{J}}_t$ does not change $\mathbf{K}^j$.

In addition to the diffusion volatility term, the two jump terms are also martingales. This is why the first term on the right side of (25), $r^j(\mathbf{n}_{t-};\mathcal{S})$, is the expected cum-dividend return. Because all firms have the same Tobin's q, the S and the U portfolios have the same shock processes and the only difference between the two portfolio is the expected return term: $r^j(\mathbf{n}_{t-};\mathcal{S})$ for the type-j portfolio. We verify these equilibrium results in Appendix A.

3.2.2 Wealth Dynamics

Let W_t denote the representative agent's wealth. Let H_t^S and H_t^U denote the dollar amount invested in the S and U portfolios, respectively. Let H_t denote the agent's wealth allocated

[15]Note that the last term in (25) is zero in the $\mathcal{B}$ state.

to sustainable and unsustainable firm equity at time t. That is, $H_t = H_t^S + H_t^U$. The dollar amount invested in the risk-free asset is then given by $(W_t - H_t)$.

In state $\mathcal{S}$, the agent's wealth evolves as:

$$dW_t = \left[r^f(\mathbf{n}_{t-};\mathcal{S})\left(W_{t-} - H_{t-}\right) - C_{t-}\right]dt + \left(r^S(\mathbf{n}_{t-};\mathcal{S})H_{t-}^S + r^U(\mathbf{n}_{t-};\mathcal{S})H_{t-}^U\right)dt + \sigma H_{t-}d\mathcal{W}_t$$
$$- \left[(1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{S})dt\right) - \frac{\mathbf{q}(\mathbf{n}_{t-};\mathcal{S}') - \mathbf{q}(\mathbf{n}_{t-};\mathcal{S})}{\mathbf{q}(\mathbf{n}_{t-};\mathcal{S})}\left(d\widetilde{\mathcal{J}}_t - \zeta(\mathbf{n}_{t-};\mathcal{S})dt\right)\right]H_{t-}\,. \tag{26}$$

The first term in (26) is the interest income from savings in the risk-free asset minus consumption. The second term is the expected capital gains from investing in the S and U portfolios. Note that the expected returns are different: $r^S(\mathbf{n};\mathcal{S})$ and $r^U(\mathbf{n};\mathcal{S})$ for the S and U portfolios, respectively. The third and fourth terms contain the diffusion and two jump martingales for the stock market portfolio. This is because the stochastic components of the returns (diffusion and jumps) for the S and U portfolios are identical path by path.[16]

In equilibrium, the dollar allocation to the S portfolio (H_t^S), as a fraction of the agent's total dollar allocations to the risky assets ($H_t = H_t^S + H_t^U$), $\pi_t^S = H_t^S/H_t = H_t^S/W_t$, equals the fraction of aggregate wealth mandated for investment in the S portfolio: $\pi^S = \alpha$. The remaining $1-\pi^S$ fraction of H_t is allocated to the U portfolio. That is, we have $H_t^S = \alpha W_t = \mathbf{Q}_t^S = \alpha\mathbf{Q}_t$, $H_t^U = (1-\alpha)W_t = \mathbf{Q}_t^U = (1-\alpha)\mathbf{Q}_t$, and $W_t = \mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U$.

Let $V_t = V(W_t, \mathbf{n}_t; \mathcal{S}_t)$ denote the agent's value function. The HJB equation for the value function in state $\mathcal{S}$, $V(W,\mathbf{n};\mathcal{S})$, satisfies (see Appendix A.1 for details):

$$0 = \max_C\ f(C,V;\mathcal{S}) + \left[\left(r^S(\mathbf{n};\mathcal{S})\alpha + r^U(\mathbf{n};\mathcal{S})(1-\alpha)\right)W - C + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))W\right]V_W$$
$$+\zeta(\mathbf{n};S)\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}WV_W + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}V_{\mathbf{n}} + \frac{\sigma^2W^2V_{WW}}{2}$$
$$+\lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[V\left(ZW,\mathbf{n};\mathcal{S}\right) - V(W,\mathbf{n};\mathcal{S})\right]$$
$$+\zeta(\mathbf{n};S)\left[V\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}W,\mathbf{n};\mathcal{S}'\right) - V(W,\mathbf{n};\mathcal{S})\right]. \tag{27}$$

The FOC for consumption C in both climate states is given by the following FOC:

$$f_C(C,V;\mathcal{S}) = V_W(W,\mathbf{n};\mathcal{S})\,. \tag{28}$$

[16]Here, to ease exposition, we use the equilibrium result that all firms have the same average q in equilibrium, which we show in Proposition 1 in the Subsection 3.3.

This is the standard consumption FOC for recursive utility. We can show that the value function $V(W,\mathbf{n};\mathcal{S})$ is homogeneous with degree $1-\gamma$ in W:

$$V(W,\mathbf{n};\mathcal{S}) = \frac{1}{1-\gamma}(u(\mathbf{n};\mathcal{S})W)^{1-\gamma}\,, \tag{29}$$

where $u(\mathbf{n};\mathcal{S})$ is a welfare measure proportional to the representative agent's equilibrium certainty equivalent wealth to be determined. Substituting (29) into the FOC (28) yields the following linear consumption rule with a time-varying MPC that depends on $\mathbf{n}$ and $\mathcal{S}$:[17]

$$C(W,\mathbf{n};\mathcal{S}) = \rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi}W\,. \tag{30}$$

Substituting (30) and (29) into the HJB equation (27), we obtain the following ODE for $u(\mathbf{n};\mathcal{S})$ in state $\mathcal{S}$:

$$\begin{aligned} 0 \;=\; & \frac{\rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi}-\rho}{1-\psi^{-1}} + \alpha r^{S}(\mathbf{n};\mathcal{S}) + (1-\alpha)r^{U}(\mathbf{n};\mathcal{S}) - \rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi} + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S}')/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}'))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ & + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')q(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})q(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{31}$$

Note that the last two terms are only present in state $\mathcal{G}$ but not state $\mathcal{B}$. The reason is that a stochastic transition only occurs from state $\mathcal{G}$ to state $\mathcal{B}$, as $\mathcal{B}$ is an absorbing state.[18]

3.3 Market Equilibrium

The equilibrium risk-free rate (r_t^f), the expected returns for the S and U portfolios (r_t^S and r_t^U), and Tobin's average q (q_t) for all firms are functions of $\mathbf{n}_t$ given the climate state $\mathcal{S}_t$. For brevity, whenever causing no confusion, we suppress the dependence on the climate state $\mathcal{S}$.

Proposition 1 summarizes equilibrium outcomes for S versus U firms.

[17] Since our model is a representative-agent framework, the aggregate financial wealth, $\mathbf{W}_t$, is equal to W_t for all t. We thus simply use these two interchangeably. See Pindyck and Wang (2013) for a similar consumption rule in their model.

[18] We first solve the ODE for climate state $\mathcal{B}$ and then solve the ODE for climate state $\mathcal{G}$ using the equilibrium objects in state $\mathcal{B}$.

Proposition 1 *For a given scaled aggregate decarbonization capital stock* $\mathbf{n}$ *and the climate state* $\mathcal{S}$*, all firms have the same Tobin's average* q*, which in equilibrium is also Tobin's average* q *for the aggregate economy (*$\mathbf{q}$*):*

$$q^S(\mathbf{n};\mathcal{S}) = q^U(\mathbf{n};\mathcal{S}) = \mathbf{q}(\mathbf{n};\mathcal{S})\,. \tag{32}$$

The investment-capital ratio for all firms is the same and equal to the aggregate investment-capital ratio $\mathbf{i}(\mathbf{n};\mathcal{S})$*:*

$$i^S(\mathbf{n};\mathcal{S}) = i^U(\mathbf{n};\mathcal{S}) = \mathbf{i}(\mathbf{n};\mathcal{S})\,. \tag{33}$$

The investment-q equation also holds at the aggregate: $\mathbf{q}(\mathbf{n};\mathcal{S}) = \frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))}$*. The cash-flow wedge between a* U *and an* S *firm equals the firm's mandated mitigation spending* $m(\mathbf{n};\mathcal{S})$*:*

$$cf^U(\mathbf{n};\mathcal{S}) - cf^S(\mathbf{n};\mathcal{S}) = m(\mathbf{n};\mathcal{S})\,, \tag{34}$$

where $cf^U(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S})$ *is the scaled cash flow for a* U *firm.*

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. Hence, all firms have the same Tobin's average q. Equations (23) and (32) imply that all firms must also have the same investment-capital ratio.

3.3.1 Cash-flow Wedge and Required Rate of Return Wedge

Importantly, U firms generate more free cash flows and hence pay more dividends to shareholders. How can the two types of firms have the same market valuation (Tobin's q) when one type pays more dividends than the other? This is because U firms that pay more dividends also have to compensate investors with higher expected rates of returns than S firms. Next, we summarize the required rate of return wedge between S and U firms.

Proposition 2 *Given that the sustainable firm spends* $m(\mathbf{n};\mathcal{S})$ *for each unit of its productive capital on mitigation, the required rate of return wedge between a* U *and an* S *firm is given by*

$$r^U(\mathbf{n};\mathcal{S}) - r^S(\mathbf{n};\mathcal{S}) = \frac{m(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,. \tag{35}$$

That is, by being sustainable, a firm lowers its required rate of return from $r^U(\mathbf{n};\mathcal{S})$ to $r^S(\mathbf{n};\mathcal{S})$ by $\frac{m(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}$. This is one of the key predictions of our model.

3.3.2 Scaled Aggregate Mitigation, Investment, and Consumption

The next proposition summarizes the results for $\mathbf{x}(\mathbf{n};\mathcal{S})$, $\mathbf{i}(\mathbf{n};\mathcal{S})$, and $\mathbf{c}(\mathbf{n};\mathcal{S})$.

Proposition 3 *The relation between the firm-level (scaled) mitigation spending $m(\mathbf{n};\mathcal{S})$ and the aggregate (scaled) mitigation spending $\mathbf{x}(\mathbf{n};\mathcal{S}) = \mathbf{X}(\mathbf{n};\mathcal{S})/\mathbf{K}$ is given by:*[19]

$$m(\mathbf{n};\mathcal{S}) = \frac{\mathbf{x}(\mathbf{n};\mathcal{S})}{\alpha} \geq \mathbf{x}(\mathbf{n};\mathcal{S})\,. \tag{36}$$

The aggregate investment-capital ratio $\mathbf{i}(\mathbf{n};\mathcal{S})$ satisfies:

$$\begin{aligned} 0 \;\; = \;\; & \frac{(A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S}))\,\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))-\rho}{1-\psi^{-1}} + \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ & + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} - \frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{S})+\mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{S})}{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}\right) \\ & + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{(A-\mathbf{i}(\mathbf{n};\mathcal{S}')-\mathbf{x}(\mathbf{n};\mathcal{S}'))\mathbf{q}(\mathbf{n};\mathcal{S})^{\psi}}{(A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S}))\mathbf{q}(\mathbf{n};\mathcal{S}')^{\psi}}\right)^{\frac{1-\gamma}{1-\psi}} - 1\right]\,. \end{aligned} \tag{37}$$

The aggregate (scaled) consumption $\mathbf{c}(\mathbf{n};\mathcal{S})$ is equal to the aggregate (scaled) dividend $\mathbf{cf}(\mathbf{n};\mathcal{S})$:

$$\mathbf{c}(\mathbf{n};\mathcal{S}) = \mathbf{cf}(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})\,. \tag{38}$$

Since each S firm spends $m(\mathbf{n}_t;\mathcal{S}_t)K_t^S$ units on mitigation and all firms are the same, the mitigation spending mandate for a firm, $m(\mathbf{n};\mathcal{S})$, is $1/\alpha$ times the aggregate scaled mitigation, $\mathbf{x}(\mathbf{n};\mathcal{S})$, where α is the fraction of S firms in equilibrium (see equation (36)). The last term in (37) captures the effect of climate-state transition on the aggregate investment ratio $\mathbf{i}(\mathbf{n};\mathcal{S})$. The aggregate consumption equals the aggregate dividend, which is the residual cash flows from operations after we subtract the aggregate investment and mitigation spending.

3.3.3 Aggregate Tobin's Average q, $\mathbf{q}(\mathbf{n};\mathcal{S})$, and Asset Pricing Implications

In the next proposition, we summarize the key asset-market predictions in the economy.

Proposition 4 *Tobin's q for the aggregate economy, q, $\mathbf{q}(\mathbf{n};\mathcal{S})$, and the aggregate investment, $\mathbf{i}(\mathbf{n};\mathcal{S})$, satisfy the same equation as the investment-q relation at the firm level:*

$$\mathbf{q}(\mathbf{n};\mathcal{S}) = \frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))}\,. \tag{39}$$

[19]This is the case provided that the firm-level mitigation spending is feasible in that $m(\mathbf{n};\mathcal{S})$ can be funded.

The aggregate stock-market risk premium in state $\mathcal{S}$, $r^M(\mathbf{n};\mathcal{S}) - r^f(\mathbf{n};\mathcal{S})$, is given by

$$r^M(\mathbf{n};\mathcal{S}) - r^f(\mathbf{n};\mathcal{S}) = \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right] + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{-\gamma} - 1\right]. \quad (40)$$

The equilibrium interest rate in state $\mathcal{S}$, $r^f(\mathbf{n};\mathcal{S})$, is given by

$$r^f(\mathbf{n};\mathcal{S}) = \frac{\mathbf{c}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \gamma\sigma^2 + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} - \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[(1-Z)Z^{-\gamma}\right] - \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{-\gamma}. \quad (41)$$

While the Tobin's q result for the aggregate economy is similar to the standard FOC for corporate investment as in the q-theory literature, this result in our model is an outcome of both individual firm's optimization and market clearing. The equilibrium market risk premium and interest rate formulas generalize those in Pindyck and Wang (2013) and Hong, Wang, and Yang (2022) by incorporating the effect of decarbonization capital stock and the climate transition risk. The first term on the right side of (40) is the standard diffusion shock contribution to the equity risk premium. The second term is the weather-disaster-shock contribution to the equity risk premium. The third term, which only exists in state $\mathcal{G}$, is the risk premium due to the stochastic tipping-point arrival.

Similarly, the last two terms on the right side of (41) for the risk-free rate $r^f(\mathbf{n};\mathcal{S})$ capture the effects of weather-disaster and climate-transition shocks on $r^f(\mathbf{n};\mathcal{S})$. The fourth term in (41) captures the effect of decarbonization capital accumulation and the first three terms are the standard terms (due to dividends, productive capital accumulation, and diffusion shocks) on $r^f(\mathbf{n};\mathcal{S})$ as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2022).

3.4 Market Economy with the Welfare-Maximizing Mandate

For a given level of α, we endogenize the qualification standard, characterized by the mitigation threshold $M_t = m(\mathbf{n_t};\mathcal{S}_t)K_t$, for a firm to be sustainable. To be precise, at time 0, the planner announces the criterion $\{M_t;\, t \geq 0\}$ and commits to the policy with the goal of maximizing the representative agent's utility given in (13). The representative agent and firms optimize taking the planner's mandate as given.[20]

Consider the agent's optimization problem. First, the homogeneity property of our model implies that the agent's value function is homogeneous of degree $1-\gamma$ in wealth W. Second, in equilibrium the agent's wealth is all invested in the stock market and therefore W is proportional to the aggregate capital stock $\mathbf{K}$. Taking these two observations together, we may write the agent's value function as follows:

$$J(\mathbf{K},\mathbf{N};\mathcal{S}) = V(W,\mathbf{n};\mathcal{S}) = \frac{1}{1-\gamma}(b(\mathbf{n};\mathcal{S})\mathbf{K})^{1-\gamma}\,, \tag{42}$$

where $b(\mathbf{n};\mathcal{S})$ is a welfare measure given by

$$b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times \mathbf{q}(\mathbf{n};\mathcal{S})\,. \tag{43}$$

For brevity, we suppress $\mathcal{S}$ whenever doing so causes no confusion. Equation (43) follows from the equilibrium result that $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ as all households' wealth is in the stock market, which is valued at $\mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$. Substituting $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ into the agent's value function $V(W,\mathbf{n};\mathcal{S})$ given in (29) for the market economy yields $J(\mathbf{K},\mathbf{N};\mathcal{S})$ given in (42) and (43). Note that $b(\mathbf{n};\mathcal{S})$ equals the product of $u(\mathbf{n};\mathcal{S})$ appearing in the agent's objective (29) and the equilibrium (aggregate) Tobin's q, $\mathbf{q}(\mathbf{n};\mathcal{S})$. That is, $b(\mathbf{n};\mathcal{S})$ captures information from both the agent's and the representative firm's optimization problems. The function $b(\mathbf{n};\mathcal{S})$ can be naturally interpreted as a welfare measure proportional to certainty equivalent wealth (scaled by the size of the economy $\mathbf{K}$).

Using the optimal consumption rule (30), the investment FOC (39), and the resource constraint $\mathbf{c}(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})$, we obtain the following equilibrium condition:

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S})\,, \tag{44}$$

which reflects information from both the firm's and the agent's optimization decisions. In

[20] Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature (e.g., Lucas and Stokey, 1983) in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.

Appendix A, we show that $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times \mathbf{q}(\mathbf{n};\mathcal{S})$ also satisfies the following ODE:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}}-1\right] + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \\ & +\phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{S}')}{b(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{45}$$

This ODE for $b(\mathbf{n};\mathcal{S})$ summarizes information about both $u(\mathbf{n};\mathcal{S})$ and $\mathbf{q}(\mathbf{n};\mathcal{S})$.

Having obtained the agent's value function and optimal policies, we turn to the planner's problem of choosing $\mathbf{x}$ to maximize $J(\mathbf{K},\mathbf{N};\mathcal{S})$ (and equivalently $b(\mathbf{n};\mathcal{S})$), which yields:

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \omega'(\mathbf{x}/\mathbf{n})b'(\mathbf{n};\mathcal{S})\,. \tag{46}$$

Let $\mathbf{n}^{ss}(\mathcal{S})$ denote the steady-state value of $\mathbf{n}$ in state $\mathcal{S}$, where the drift of $\mathbf{n}$ is zero. Therefore, by setting (10) to zero, we obtain the following relation linking aggregate investment $\mathbf{i}^{ss}(\mathcal{S})$ and mitigation spending $\mathbf{x}^{ss}(\mathcal{S})$:

$$\omega(\mathbf{x}^{ss}(\mathcal{S})/\mathbf{n}^{ss}(\mathcal{S})) - \phi(\mathbf{i}^{ss}(\mathcal{S})) = 0\,. \tag{47}$$

Additionally, substituting the zero-drift condition (47) into (45), we obtain the following equation at the steady state:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^{ss}(\mathcal{S})-\mathbf{x}^{ss}(\mathcal{S})}{b(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S})}\right)^{1-\psi^{-1}}-1\right] + \phi(\mathbf{i}^{ss}(\mathcal{S})) - \frac{\gamma\sigma^2}{2} \\ & +\frac{\lambda(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] + \frac{\zeta(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S})}{1-\gamma}\left[\left(\frac{b(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S}')}{b(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{48}$$

Solution Summary. At the steady state where $d\mathbf{n}_t = 0$, the scaled decarbonization capital stock $\mathbf{n}^{ss}(\mathcal{S})$, $\mathbf{i}^{ss}(\mathcal{S})$, $\mathbf{x}^{ss}(\mathcal{S})$, and the welfare measure $b(\mathbf{n}^{ss}(\mathcal{S});\mathcal{S})$ jointly solve the four pairs of equations (for $\mathcal{G}$ and $\mathcal{B}$): the FOC (46) for $\mathbf{x}^{ss}(\mathcal{S})$, the FOC (44) for $\mathbf{i}^{ss}(\mathcal{S})$, the zero-drift condition (47) for $\mathbf{n}^{ss}(\mathcal{S})$, and (48) for $b(\mathbf{n}^{ss};\mathcal{S})$.

For the transition dynamics, the scaled mitigation spending $\mathbf{x}_t$, the investment-capital ratio $\mathbf{i}_t$, and the welfare measure b_t are all functions of the scaled decarbonization capital stock $\mathbf{n}_t$ and the climate state $\mathcal{S}_t$. We fully characterize the solution for the transition dynamics as follows. The functions $\mathbf{x}(\mathbf{n};\mathcal{S})$, $\mathbf{i}(\mathbf{n};\mathcal{S})$, and $b(\mathbf{n};\mathcal{S})$ for both $\mathcal{G}$ and $\mathcal{B}$ states jointly solve the ODE system of the following three pairs of equations: the FOC (44) for $\mathbf{i} = \mathbf{i}(\mathbf{n};\mathcal{S})$, the FOC (46) for $\mathbf{x} = \mathbf{x}(\mathbf{n};\mathcal{S})$, and the ODE (45) for $b(\mathbf{n};\mathcal{S})$ subject to the boundary conditions at the steady state summarized above.

3.5 Comments on Competitive Market Economy with Mandates

3.5.1 Relation between α and Firm Qualification Standards m

It is worth highlighting a few key properties of our welfare-maximizing mandate. In our model, the parameter α is given. Provided that α is large enough so that a firm choosing to be sustainable can afford spending $m = \mathbf{x}/\alpha$ per unit of its capital stock on mitigation spending, the equilibrium *aggregate* mitigation spending $\mathbf{x}$ of the market economy with optimal mandates can be implemented. Note that the welfare-maximizing mandate or equivalently the qualification standard for firms to be sustainable adjusts — when α is larger, the qualification standards $m = \mathbf{x}/\alpha$ for each firm become lower since there are more firms that are sustainable. This is possible because, given the assumptions about decarbonization technology, it is irrelevant which firms, or how many, invest in decarbonization. That is, it is sufficient to have a set of firms doing all of the decarbonization capital investments, as long as the sum of their contributions allow the economy to reach the aggregate $\mathbf{X}_t$ target we set. Put differently, our market economy with an optimal mandate only pins down $\mathbf{x}_t$ or equivalently, the product of α and firm-level mitigation spending m. To pin down α and m separately would require additional information. For instance, if we knew in the data what m was, we could maximize welfare by choosing α. In Subsection 6.3, we discuss these issues in detail and solve for the model using this alternative setup.

3.5.2 Sustainable Finance Tax: Mandated Market Economy with $\alpha = 1$

What if the investment mandate requires all investors to be sustainable: $\alpha = 1$? This is in effect a sustainable finance tax where firms have no choice but to be sustainable. Our welfare-maximizing economy yields the same outcomes as an economy with capital (or equivalently sales) taxation with probability one.

Let τ_t denote the tax rate at which the government levies on each firm's capital or equivalently sales as $Y = AK$ and A is a constant. We define the competitive equilibrium with sustainable-finance taxation as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the U portfolio, S portfolio, and the risk-free asset; (2) each firm chooses its investment policy I to maximize its market value by solving (17) where the firm's cash flow at t, $CF(\mathbf{n}_t; \mathcal{S})$, is given by $CF(\mathbf{n}_t; \mathcal{S}) = AK_t - I_t(\mathbf{n}_t; \mathcal{S}) - \tau_t K_t$; and (3) all markets clear.

The government sets the tax rate on capital stock as follows:

$$\tau_t = \tau(\mathbf{n}_t; \mathcal{S}_t) = \mathbf{x}(\mathbf{n}_t; \mathcal{S}_t) \,. \tag{49}$$

As a result, this economy attains the same resource allocation as the welfare-maximizing economy with investment mandate $\alpha = 1$. The intuition is as follows. Because taxes are mandatory for all firms, using taxation, the government effectively makes all firms "sustainable." Since the government is benevolent maximizing the representative agent's welfare, it simply sets the sustainable-finance tax rate $\tau(\mathbf{n}; \mathcal{S})$ to the same aggregate mitigation spending $\mathbf{x}(\mathbf{n}; \mathcal{S})$ as in the economy with the socially optimal investment mandate.

While taxation typically distorts decisions and hence is inefficient,[21] taxation proceeds in our model allow the government to fund the accumulation of decarbonization capital, substantially reducing the weather-disaster and climate tipping-point arrival rates so that the equilibrium resource allocation with taxation is much closer to the first-best solution, which we later show.

3.5.3 Heterogenous-Agents Model: Sustainable versus Unsustainable Investors

We may also equivalently interpret our representative-agent model (with portfolio restrictions) as a model with heterogeneous agents in which some investors have sustainability investment mandates (e.g., some large asset managers and sovereign wealth funds) and others face no sustainability mandates nor preferences for being sustainable.

[21] See Chamley (1986) and Judd (1986) for seminal contributions.

Specifically, consider a model with two types of investors: S investors whose wealth constitutes an α fraction of the economy-wide total wealth and U investors who hold the remaining wealth in the economy. Importantly, the U investors are not allowed to short sale stocks issued by S firms.[22]

Since S investors are mandated to hold S stocks, they hold the entire S portfolio. Even though S firms pay fewer dividends than U firms under all circumstances, the valuation of the two types of firms are the same because U investors cannot make arbitrage profits as they are unable to short S stocks. We can also show that the equilibrium resource allocation and asset prices in this heterogeneous-agents model are the same as in our (baseline) representative-agent model.

Despite the difference in the expected return between unsustainable and sustainable firms, investors in unsustainable firms would not dominate the economy in the long run because Tobin's q for unsustainable and sustainable firms are the same. There is no feasible gains from trade and unsustainable investors cannot make arbitrage profits given the short-sales constraint. Our no-trade and equilibrium pricing reasoning is similar to that in the equilibrium asset-pricing model with agency conflicts (between the controlling and outside minority shareholders) in Albuquerque and Wang (2008). Note that the homogeneity property (in our setting with Epstein-Zin utility and geometric processes) is key for the no-trade result.

4 First-Best Solution

In this section, we summarize the first-best solution where the planner chooses aggregate $\mathbf{C}$, $\mathbf{I}$, and $\mathbf{X}$ to maximize the representative agent's utility defined in (13)-(14). Using the homogeneity property, we work with scaled variables at the aggregate level, $\mathbf{i}_t = \mathbf{I}_t/\mathbf{K}_t$, $\mathbf{x}_t = \mathbf{X}_t/\mathbf{K}_t$, and $\mathbf{c_t} = \mathbf{C}_t/\mathbf{K}_t$. In Appendix B, we show that $\mathbf{x}(\mathbf{n};\mathcal{S})$ and $\mathbf{i}(\mathbf{n};\mathcal{S})$ for both

[22] In our heterogeneous-agents model, we need to impose the short-sale constraints for the S firms' equity. Otherwise, there is no equilibrium. This is because investor can pocket the profits by taking a long position in the U firm and a short position in the S firm. This is a textbook arbitrage example as investors with this position take no risk at all but make sure profits, as S and U firms are driven by identical shocks path and path, the prices for the two types of firms are the same, but the dividends of U firms strictly dominate the dividends of S firms.

climate states ($\mathcal{S} = \mathcal{G}, \mathcal{B}$) satisfy the following simplified FOCs:

$$\rho \left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \right)^{-\psi^{-1}} + \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\mathbf{n}b'(\mathbf{n};\mathcal{S}) = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S}) , \tag{50}$$

$$\rho \left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \right)^{-\psi^{-1}} = \omega'(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})b'(\mathbf{n};\mathcal{S}) \ . \tag{51}$$

The welfare measure (proportional to certainty equivalent wealth) in state $\mathcal{S}$, $b(\mathbf{n};\mathcal{S})$, solves the following ODE:

$$\begin{aligned} 0 \ = \ & \frac{\rho}{1-\psi^{-1}} \left[\left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \right)^{1-\psi^{-1}} - 1 \right] + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) \right] \frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \\ & + \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma} \left[\left(\frac{b(\mathbf{n};\mathcal{S}')}{b(\mathbf{n};\mathcal{S})} \right)^{1-\gamma} - 1 \right] . \end{aligned} \tag{52}$$

Because state $\mathcal{B}$ is absorbing, we first solve the triple, $b(\mathbf{n};\mathcal{B})$, $\mathbf{i}(\mathbf{n};\mathcal{B})$, and $\mathbf{x}(\mathbf{n};\mathcal{B})$, for state $\mathcal{B}$ by using (50), (51), and (52). Then, we solve the triple, $b(\mathbf{n};\mathcal{G})$, $\mathbf{i}(\mathbf{n};\mathcal{G})$, and $\mathbf{x}(\mathbf{n};\mathcal{G})$, for state $\mathcal{G}$ by using (50), (51), (52), and the $b(\mathbf{n};\mathcal{B})$ solution obtained earlier.

At the first-best steady state $\mathbf{n}^{FB}(\mathcal{S})$ for both $\mathcal{G}$ and $\mathcal{B}$ states, we have

$$\omega(\mathbf{x}^{FB}(\mathcal{S})/\mathbf{n}^{FB}(\mathcal{S})) - \phi(\mathbf{i}^{FB}(\mathcal{S})) = 0 \,. \tag{53}$$

Moreover, if $\omega(\,\cdot\,) = \phi(\,\cdot\,)$, i.e., the investment efficiency functions for the two types of capital stocks are the same, the investment-capital ratio for $\mathbf{K}$ equals that for decarbonization capital $\mathbf{N}$ at $\mathbf{n}^{FB}(\mathcal{S})$. Substituting (53) into (52) yields the following steady-state condition in $\mathcal{S}$:

$$\begin{aligned} 0 \ = \ & \frac{\rho}{1-\psi^{-1}} \left[\left(\frac{A - \mathbf{i}^{FB}(\mathcal{S}) - \mathbf{x}^{FB}(\mathcal{S})}{b(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S})} \right)^{1-\psi^{-1}} - 1 \right] + \phi(\mathbf{i}^{FB}(\mathcal{S})) - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] + \frac{\zeta(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S})}{1-\gamma} \left[\left(\frac{b(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S}')}{b(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S})} \right)^{1-\gamma} - 1 \right] . \end{aligned} \tag{54}$$

Solution Summary. At the first-best steady state where $d\mathbf{n}_t = 0$, the scaled decarbonization capital stock $\mathbf{n}^{FB}(\mathcal{S})$, $\mathbf{i}^{FB}(\mathcal{S})$, $\mathbf{x}^{FB}(\mathcal{S})$, and the welfare measure $b(\mathbf{n}^{FB}(\mathcal{S});\mathcal{S})$ jointly solve the four pairs of equations (for states $\mathcal{G}$ and $\mathcal{B}$): the FOC (51) for $\mathbf{x}^{FB}(\mathcal{S})$, the FOC (50) for $\mathbf{i}^{FB}(\mathcal{S})$, the zero-drift condition (53) for $\mathbf{n}^{FB}(\mathcal{S})$, and (54) for $b(\mathbf{n}^{FB};\mathcal{S})$.

For the transition dynamics, the scaled mitigation spending $\mathbf{x}_t$, the investment-capital ratio $\mathbf{i}_t$, and the welfare measure b_t are all functions of the scaled decarbonization capital stock $\mathbf{n}_t$ and the climate state $\mathcal{S}_t$. We fully characterize the solution for the transition dynamics as follows. The $\mathbf{x}_t = \mathbf{x}(\mathbf{n};\mathcal{S})$, $\mathbf{i}_t = \mathbf{i}(\mathbf{n};\mathcal{S})$, and $b(\mathbf{n};\mathcal{S})$ processes jointly solve the ODE system of the following three pairs of equations: the FOC (50) for $\mathbf{i}(\mathbf{n};\mathcal{S})$, the FOC (51) for $\mathbf{x}(\mathbf{n};\mathcal{S})$, and the ODE (52) for $b(\mathbf{n};\mathcal{S})$ for the two states ($\mathcal{G}$ and $\mathcal{B}$) subject to the boundary conditions (for $\mathbf{n}^{FB}(\mathcal{B})$ and $\mathbf{n}^{FB}(\mathcal{G})$) at the steady state summarized above.

5 Comparing the Welfare-Maximizing Mandate Economy to First Best

We proceed in two steps in this section. First, we show in Subsection 5.1 why the market economy with the welfare-maximizing mandate cannot attain the first-best outcome. Then, we show how the planner can attain the first-best outcome by introducing an investment tax into the market economy with the welfare-maximizing mandate in Subsection 5.2. The key insight is that by optimally designing the sustainability investment mandate and setting taxes on the deviation of corporate investment (from the average level), the planner can attain the first-best by ensuring that the aggregate decarbonization capital accumulation stays on the socially efficient path at all time.

5.1 Market Economy with the Welfare-Maximizing Mandate Does Not Attain First-Best

As no private agent has incentives to spend resources to accumulate the decarbonization capital stock, our market economy is Pareto inefficient. Although using the optimal investment mandate as a function of $\mathbf{n}_t$, $\mathbf{x}_t = \mathbf{x}(\mathbf{n}_t;\mathcal{S})$, improves welfare, the planner cannot attain the first-best by solely relying on the optimal mandate. Why? First recall that in a static economy with one source of market failure, the planner can attain the first-best by imposing the optimal Pigouvian tax to fund the first-best mitigation spending (or equivalently decarbonization capital stock). This is because the private sectors' incentives are fully aligned with the planner's once the optimal Pigouvian tax is chosen in a *static* setting.[23]

However, this simple one-instrument-for-one-market-failure argument in a static setting is invalid in our dynamic model. This is because the planner needs to choose the socially optimal $\mathbf{X}_t^{FB}$ at all time t and for all contingencies, which is an infinite dimensional problem.

A priori, there is no reason why the optimal sustainability investment mandate $\mathbf{x}_t = \mathbf{x}(\mathbf{n}_t; \mathcal{S})$ in a dynamic setting allows the planner to attain the first-best. Next, we lay out the specific differences between the mandated market economy and the planner's first-best economy.

5.1.1 Investment Distortions

By comparing the solution for the mandated market economy given in (44)-(46) with the solution for the planner's economy given in (50)-(52), we see that the different resource allocations in the two economies arise from different investment ($\mathbf{I}$) functions. Specifically, the $\mathbf{i}(\mathbf{n}; \mathcal{S})$ equation (44) for the mandated market economy is different from the $\mathbf{i}(\mathbf{n}; \mathcal{S})$ equation (50) in the first-best economy. Why does this difference exist given optimal investment mandates in the market economy? We answer this question in two steps.

First consider the planner's problem. Increasing investment $\mathbf{I}$ has two effects at the aggregate level: (1) a direct effect of reducing the resources for the representative household's consumption as $\mathbf{I}$ crowds out $\mathbf{C} = \mathbf{Y} - \mathbf{I} - \mathbf{X}$, captured by the first term on the left side of (50); and (2) an indirect effect of decreasing the scaled aggregate decarbonization capital stock, $\mathbf{n} = \mathbf{N}/\mathbf{K}$, in the future as the firm's future $\mathbf{K}$ is higher due to current investment. The latter effect is captured by the second term on the left side of (50).

In contrast, firms in the mandated market economy do not take the indirect long-term effect of investment on future $\mathbf{n}$ into account. Indeed, this second term on the left side of (50) is absent in the investment equation (44) in the mandated market economy.

The indirect long-run effect of investment on $\mathbf{n}$ in the planner's economy makes invest-

[23]Taking the optimal tax as given, the representative agent's and the firm's own incentives give rise to the first-best tradeoff between consumption and investment in the marketplace. This is the main argument underpinning the calculation for the social cost of carbon in the climate economics literature.

ment more costly than in the mandated market economy, *ceteris paribus*. By comparing (44) and (50), we can conclude that $\mathbf{i}(\mathbf{n}_t; \mathcal{S}_t)$ in the first-best economy is lower than in the mandated market economy.[24]

In sum, firms have incentives to spend more on investment in a market economy (even with optimal mandates) than socially desirable. That is, our mandated economy still features over-investment compared with the first-best. This is because the planner takes both direct and indirects costs of investing into account while firms in (mandated) market economies only take the direct effect of investing on $\mathbf{n}$ into account.

With both (direct and indirect) effects of a firm's investment on equilibrium resource allocations, we at least need two instruments to attain the first-best outcome. Next, we show that two optimally chosen instruments are sufficient to attain the first-best: one to collect proceeds from investors to fund socially desirable first-best aggregate mitigation spending and the other for the society to eliminate firms' incentives to over-accumulate capital, which in turn implies that the society can ensure that the $\mathbf{n}_t$ process follows the first-best trajectory.

5.2 Restoring First Best

To achieve the first-best in the market economy of our model, it is necessary and sufficient that at all t and for all $\mathbf{n}_t$ and $\mathcal{S}_t$ the following two conditions hold: (a.) each firm chooses the first-best investment policy at all time and (b.) the society as a whole collects resources to fund the first-best aggregate mitigation spending.

To ease exposition, we first show how to attain the first-best by properly using the following two state-contingent instruments: (1.) taxing all firms at the rate of $\tau_t = \mathbf{x}_t$ introduced in Subsection 3.5 for each unit of capital and (2.) taxing a firm's investment if its investment-capital ratio i_t exceeds the economy-wide $\mathbf{i}_t$.

Then, we can conclude that the first-best outcome can also be attained by using a combination of a sustainability mandate $m_t = \mathbf{x}_t/\alpha$ and a tax on the wedge between a firm's investment and the economy-wide $\mathbf{i}_t$. This follows from our result on the equivalence be-

[24] We use the property $b'(\mathbf{n}; \mathcal{S}) > 0$, the investment optimality (FOCs), and the concavity of $\phi(\cdot)$ (equivalently convex adjustment costs).

tween taxing a firm's capital at the rate of aggregate mitigation spending $\mathbf{x}_t$ and using a qualification standard $m_t = \mathbf{x}_t/\alpha$ for firms to be sustainable.[25]

The capital tax instrument is to fund the first-best mitigation spending $\mathbf{X}_t$. The second instrument eliminates firms' incentives to over-invest in their own capital stocks so that the aggregate productive capital accumulation $\mathbf{K}$ and decarbonization $\mathbf{N}$ follow the first-best trajectories. To achieve this goal, we tax firm j at the rate of $\widehat{\tau}^j(\mathbf{n}_t;\mathcal{S}_t)K_t^j$, where

$$\widehat{\tau}^j(\mathbf{n};\mathcal{S}) = \left[\phi(i^j) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{q}(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\,. \tag{55}$$

In (55), $\mathbf{i}(\mathbf{n};\mathcal{S})$ and $\mathbf{q}(\mathbf{n};\mathcal{S})$ are the equilibrium aggregate investment-capital ratio $\mathbf{i}$ and average $\mathbf{q}$ in state $\mathcal{S}$, respectively, and $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times \mathbf{q}(\mathbf{n};\mathcal{S})$ is a measure of welfare (proportional to the household's certainty equivalent wealth). The only term in (55) that firm j chooses is $\phi(i^j)$. If its investment i^j exceeds the economy-wide average $\mathbf{i}$, firm j pays a tax $\widehat{\tau}^j(\mathbf{n};\mathcal{S})$ given in (55) for each unit of its capital. This tax discourages corporate investment, mitigating over-investment in $\mathbf{K}$ and under-investment in $\mathbf{N}$. The multiple $\mathbf{q}(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}$ for the wedge $[\phi(i^j) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]$ in (55) is necessary to attain the first-best outcome.

Below we further explore our model's mechanism by highlighting a few key equations in our proof. First, firm j's average q, $q^j(\mathbf{n};\mathcal{S})$, satisfies the following ODE in state $\mathcal{S}$:

$$\begin{aligned} r^j(\mathbf{n};\mathcal{S})q^j(\mathbf{n};\mathcal{S}) = \max_{i^j}\; & cf^j(\mathbf{n};\mathcal{S}) - \widehat{\tau}^j(\mathbf{n};\mathcal{S}) + (\phi(i^j) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)))q^j(\mathbf{n};\mathcal{S}) \\ & + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{n}q^j_{\mathbf{n}}(\mathbf{n};\mathcal{S}) + \zeta(\mathbf{n};\mathcal{S})(q^j(\mathbf{n};\mathcal{S}') - q^j(\mathbf{n};\mathcal{S}))\,, \end{aligned} \tag{56}$$

where $\widehat{\tau}^j(\mathbf{n};\mathcal{S})$ is given in (55).

Importantly, using (55) for $\widehat{\tau}^j(\mathbf{n};\mathcal{S})$, we obtain the following investment FOC for firm j:

$$1 = \phi'(i^j)\left(q^j - \mathbf{q}(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)\,. \tag{57}$$

Then using the equilibrium results $i^U = i^S = \mathbf{i}$ and $q^U = q^S = \mathbf{q}$, we obtain the following equation for the aggregate investment-capital ratio $\mathbf{i}$:

$$\rho\left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} + \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\mathbf{n}b'(\mathbf{n};\mathcal{S}) = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S})\,, \tag{58}$$

[25]This equilvalence result holds provided that α is large enough so that $m = \mathbf{x}/\alpha$ is feasible at the firm level given its resource constraint (as discussed in Subsection 3.5).

which is the same as the FOC (50) in the first-best economy. The second term on the left side of (58) arises from the formula for the tax rate $\widehat{\tau}^j(\mathbf{n};\mathcal{S})$, which depends on the investment wedge $[\phi(i^j) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]$. This investment wedge tax allows us to attain the first-best outcome. For brevity, we relegate some details of the proofs (e.g., verifying the value functions, policy functions, and the equivalence between the two implementations) to Appendix C.2.

6 Quantitative Analysis

In this section, we calibrate our model to study how well mandates approximate the first-best. We focus on the parameter region where the planner chooses to act now to decarbonize—that is, where the planner makes significant annual mitigation spending contributions and smoothly ramp up to a high steady-state decarbonization-to-productive capital ratio $\mathbf{n}^{ss}$.

6.1 Functional Form Specifications

We begin by specifying various functional forms in our model.

6.1.1 Firm-level Capital K and Aggregate Decarbonization Capital $\mathbf{N}$

As in Pindyck and Wang (2013), we specify a firm's investment-efficiency function $\phi(i)$ as

$$\phi(i) = i - \frac{\eta_K \, i^2}{2} - \delta_K \,, \tag{59}$$

where η_K measures the degree of adjustment costs and δ_K is a constant that can be viewed as the depreciation rate.

Similarly, at the aggregate level, we assume that the controlled drift for the aggregate decarbonization capital stock $\mathbf{N}$ takes the same form as that for firm-level capital stock K:

$$\omega(\mathbf{x}/\mathbf{n}) = (\mathbf{x}/\mathbf{n}) - \frac{\eta_{\mathbf{N}} \, (\mathbf{x}/\mathbf{n})^2}{2} - \delta_{\mathbf{N}} \,, \tag{60}$$

where $\eta_{\mathbf{N}}$ is the adjustment cost parameter for the aggregate decarbonization capital $\mathbf{N}$. Note that $\mathbf{x}/\mathbf{n} = \mathbf{X}/\mathbf{N}$ is the aggregate investment $\mathbf{X}$ in the decarbonization capital scaled by $\mathbf{N}$, which is analogous to a firm's investment level scaled by its capital stock: $i = I/K$.

Delaying the tipping point arrival. By accumulating decarbonization capital stock, the society decreases the tipping-point arrival rate from $\zeta_0 > 0$ to

$$\zeta(\mathbf{n};\mathcal{G}) = \zeta_0(1-\mathbf{n}^{\zeta_1})\,, \tag{61}$$

where $0 < \zeta_1 < 1$. (Recall that $\zeta(\mathbf{n};\mathcal{B}) = 0$.) For a given $\mathbf{n}$, the lower the value of ζ_1 the more efficient the decarbonization capital stock is at curtailing the tipping-point arrival.[26]

6.1.2 Conditional Damage and Weather-disaster Arrival Rates

In a given climate state $\mathcal{S}$, decarbonization capital $\mathbf{N}$ can also ameliorate the damage to economic growth by reducing the frequencies of weather-disaster (e.g., high-temperature) events. Specifically, we use the following specification for the weather-disaster arrival rate $\lambda(\mathbf{n};\mathcal{S})$ in state $\mathcal{S}$:

$$\lambda(\mathbf{n};\mathcal{S}) = \lambda_0^{\mathcal{S}}(1-\mathbf{n}^{\lambda_1})\,, \tag{62}$$

where $\lambda_0^{\mathcal{S}} > 0$ is the arrival rate absent any decarbonization capital stock ($\mathbf{n} = 0$) in climate state $\mathcal{S}$ and $\lambda_1 \in (0,1)$ measures how efficient the aggregate decarbonization capital stock reduces the weather-disaster arrival rate $\lambda(\mathbf{n};\mathcal{S})$. For brevity, we assume that λ_1 is the same in the two climates states $\mathcal{G}$ and $\mathcal{B}$. Similar to the effect of ζ_1 on the tipping-point arrival, a lower value of λ_1 is associated with a more efficient decarbonization technology reducing the weather disaster arrival rate, *ceteris paribus.*

The expected aggregate growth rate in state $\mathcal{S}$, $\mathbf{g}(\mathbf{n};\mathcal{S})$, is

$$\mathbf{g}(\mathbf{n};\mathcal{S}) = \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \lambda(\mathbf{n};\mathcal{S})\ell + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,, \tag{63}$$

where ℓ, the expected fractional capital loss conditional on a jump arrival, is given by

$$\ell = \mathbb{E}(1-Z) = \frac{1}{\beta+1}\,. \tag{64}$$

Note that a lower value of β is associated with a more damaging and also more fat tailed disaster. The first term in (63), $\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))$, is the expected growth in state $\mathcal{S}$ absent jumps

[26] This follows from $\partial\zeta(\mathbf{n};\mathcal{G})/\partial\zeta_1 = -\zeta_0\mathbf{n}^{\zeta_1}\ln(\mathbf{n}) > 0$ as $\mathbf{n} < 1$.

and the second term adjusts for the effect of weather-disaster arrivals. The last term in (63) captures the effect of the climate-state transition from state $\mathcal{G}$ to $\mathcal{B}$ on the expected growth rate in state $\mathcal{G}$. Finally, the last term is zero in state $\mathcal{B}$ as state $\mathcal{B}$ is absorbing: $\zeta(\mathbf{n}; \mathcal{B}) = 0$.

6.2 Baseline Calibration

Our model has fifteen parameters in total. We next choose parameter values based on known key macro-finance moments and empirical studies on climate mitigation pathways involving decarbonization. Our calibration exercise is intended to highlight the extent to which mandates can approximate the social planner's solution when the planner wants to act now to decarbonize the economy. We summarize the values of these parameters for our baseline analysis in Table 1.

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	4.2%
coefficient of relative risk aversion	γ	8
productivity for K	A	26%
adjustment cost parameter for K	η_K	5
adjustment cost parameter for $\mathbf{N}$	$\eta_{\mathbf{N}}$	5
diffusion volatility for $\mathbf{N}$ and K	σ	9%
depreciation rates for $\mathbf{N}$ and K	$\delta_K = \delta_{\mathbf{N}}$	6%
jump arrival baseline parameter from state $\mathcal{G}$ to $\mathcal{B}$	ζ_0	0.02
jump arrival sensitivity parameter from state $\mathcal{G}$ to $\mathcal{B}$	ζ_1	0.1
power-law exponent	β	39
jump arrival baseline parameter with $\mathbf{n} = 0$ in state $\mathcal{G}$	$\lambda_0^{\mathcal{G}}$	0.05
jump arrival baseline parameter with $\mathbf{n} = 0$ in state $\mathcal{B}$	$\lambda_0^{\mathcal{B}}$	2
mitigation technology parameter	λ_1	0.3

All parameter values, whenever applicable, are continuously compounded and annualized.

6.2.1 Preferences Parameters

We choose a value for the time rate of preferences within the standard range: $\rho = 4.2\%$ per annum. We set the coefficient of relative risk aversion at $\gamma = 8$ and the EIS at $\psi = 1.5$, both of which are within the standard ranges used in the long-run risk literature (Bansal and Yaron, 2004).[27]

6.2.2 Parameters for Productive and Decarbonization Capital

We set the productivity parameter $A = 26\%$ per annum and the capital adjustment parameter $\eta_K = 5$ to target an average q of 2.5 and an average growth rate of 2.2% per annum in the pre-climate-change sample. The values of $A = 26\%$ and $\eta_K = 5$ are within the range of empirical estimates (Stokey and Rebelo, 1995; Eberly, Rebelo, and Vincent, 2012). Decarbonization capital has no productivity but faces adjustment costs as physical capital. We set the decarbonization capital adjustment cost parameter $\eta_{\mathbf{N}} = \eta_K = 5$ for parsimony and also under the premise that direct air capture and plants are themselves a form of physical capital. We set the annual diffusion volatility at $\sigma = 9\%$ (Pindyck and Wang, 2013) to target a historical stock market risk premium of about 6% per annum (Hansen and Singleton, 1982; Mehra and Prescott, 1985). The annual depreciation rate for productive $\delta_K = 6\%$, is in line with the literature cited above as well. Again, for parsimony, we set $\delta_{\mathbf{N}} = \delta_K$.

6.2.3 Parameters for Delaying the Tipping Point of Climate Transition

Recent studies indicate that tipping points in the climate system can occur even at current levels of warming (Lenton et al., 2019). To generate a sizeable act-now effect, we set the expected arrival rate of a tipping point to be once every 50 years: $\zeta_0 = 0.02$. We then build on estimates from Gates (2021) who proposes that spending around \$5 trillion dollars each year on carbon capture can forever eliminate the problem of global warming (this estimate

[27] Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero to values as high as two. Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Guvenen (2006) reconciles the conflicting evidence on the elasticity of intertemporal substitution from a macro perspective. In the long-run risk literature, it is critical to choose an EIS value larger than one.

is based on \$100 per ton cost of capture and there are 51 billion tons of carbon emissions per year).[28] We consider a more modest scenario similar to de Pee et al. (2018) where spending a couple of trillion dollars per year on decarbonizing heavy industries can substantially reduce the tipping point arrival rate from 2% per annum ($1/\zeta_0 = 50$) to around 0.5% per annum (with an implied expected arrival in 200 years). This calibration yields a value of $\zeta_1 = 0.1$.[29]

6.2.4 Parameters for Weather Disasters and Conditional Damage Functions

Since weather disasters, e.g., droughts, are associated with high temperatures, we calibrate the parameter $\lambda_0^{\mathcal{G}}$ describing the arrival rate of weather disasters in state $\mathcal{G}$ and the parameter β measuring the expected damages conditional on arrival, $\ell = (\beta + 1)^{-1}$, using a set of panel regressions documenting the adverse effects of weather shocks in the form of extreme temperatures for economic growth (Dell, Jones, and Olken, 2012).[30]

First, we calibrate β as follows. For the median country in the Dell, Jones, and Olken (2012) sample, extreme weather disasters in the form of extremely high temperatures lowers the GDP growth rate by 2.5% per annum. To match this moment, we set $\beta = 39$ as the implied reduction of GDP growth conditional on a disaster arrival is $\ell = 1/(\beta+1) = 1/40 = 2.5\%$ per annum. Second, using again the Dell, Jones, and Olken (2012) sample, we infer that the weather disaster arrival rate in state $\mathcal{G}$ is low: around $\lambda_0^{\mathcal{G}} = 0.05$ per annum in the pre-climate-change sample. In other words, such weather disaster events are uncommon, occurring in five percent of the country-year (annual) observations. Our analysis is most apt for the median country in their sample. But our model can be recalibrated for any subset of countries. For state $\mathcal{B}$, we set $\lambda_0^{\mathcal{B}} = 2$, a forty times increase in weather disaster frequencies,

[28]Reforestation also has the potential to contribute to keeping global temperatures from breaching the 1.5^o Celsius barrier. This adjustment process is also expensive like building direct air capture plants (Bastin et al. (2019) and Griscom et al. (2017)).

[29]At the steady state in $\mathcal{G}$, $\zeta(\mathbf{n}^{ss};\mathcal{G}) = \zeta_0(1-(\mathbf{n}^{ss})^{\zeta_1}) = 0.02 \times (1 - 6.13\%^{0.1}) \approx 205$ years.

[30]This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts, 2009) by using location and time fixed effects. But it is now applied to many other contexts including economic growth and productivity. The main idea is that abnormally high annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output. Burke, Hsiang, and Miguel (2015) find that the effects of temperature on growth is nonlinear. But we stay with the linear specification of Dell, Jones, and Olken (2012).

following studies of tipping points cited in the Introduction. Third, we set $\lambda_1 = 0.3$ for the arrival rate $\lambda(\mathbf{n}; \mathcal{S})$ in both states $\mathcal{G}$ and $\mathcal{B}$ so that the decarbonization-to-productive capital ratio $\mathbf{n}$, which lowers temperatures, not only delays the tipping-point arrival also reduces the frequency of weather disasters, as is often modeled in climate science and integrated assessment models.

6.3 Comparing Laissez Faire, Markets with Optimal Mandate, and First-Best Economies

We first provide a quantitative comparison across the steady-state solutions for the three economies: laissez faire, market economy with welfare-maximizing mandate, and first-best.

Table 2: COMPARING ACROSS THE LAISSEZ FAIRE, THE MANDATED MARKET, AND THE FIRST-BEST ECONOMIES IN STATE $\mathcal{G}$. The steady-state value of $\mathbf{n}$ in state $\mathcal{G}$ is $\mathbf{n}^{ss} = 0.0613$.

		laissez faire	mandate	first-best
scaled mitigation spending	$\mathbf{x}^{ss}$	0	0.76%	0.78%
scaled decarbonization stock	$\mathbf{n}^{ss}$	0	6.13%	6.48%
scaled aggregate investment	$\mathbf{i}^{ss}$	11.83%	12.41%	12.07%
Tobin's average q	$\mathbf{q}^{ss}$	2.45	2.64	2.52
scaled aggregate consumption	$\mathbf{c}^{ss}$	14.17%	12.82%	13.15%
expected GDP growth rate	$\mathbf{g}^{ss}$	2.04%	2.44%	2.30%
(real) risk-free rate	$r^{f,ss}$	1.10%	0.73%	0.91%
stock market risk premium	rp^{ss}	6.73%	6.58%	6.60%
aggregate welfare measure	b^{ss}	0.0542	0.0826	0.0830
time from $\mathbf{n} = 0$ to $0.99\mathbf{n}^{ss}$ in $\mathcal{G}$		0	10.9	10.0

6.3.1 Comparing Steady-state Solutions

We summarize the steady-state results in state $\mathcal{G}$ in the three columns of Table 2. The column labeled "laissez faire" reports the results for the laissez faire economy (i.e., $\alpha = 0$). The column labeled "mandate" reports the solution for the mandated market economy and the column labeled "first-best" reports the first-best solution.

In the laissez faire economy, as firms have no incentives to provide public goods (aggregate risk mitigation), there is no mitigation spending ($\mathbf{x} = 0$) and thus $\mathbf{n}^{ss} = 0$. In the market economy with optimal investment mandates, the aggregate decarbonization capital stock is $\mathbf{n}^{ss} = 6.13\%$, which falls only slightly short of the first-best level: $\mathbf{n}^{FB} = 6.48\%$. The annual contribution of mitigation spending is also only slightly under the first-best: $\mathbf{x}^{ss} = 0.76\% < \mathbf{x}^{FB} = 0.78\%$.

Also note that firms facing optimal mandates still over-invest in capital accumulation compared to the first-best: $\mathbf{i}^{ss} = 12.41\% > \mathbf{i}^{FB} = 12.07\%$. In contrast and as expected, firms under-invest in the laissez faire economy compared to first-best: $\mathbf{i}^{ss} = 11.83\% < \mathbf{i}^{FB} = 12.07\%$, as the laissez faire economy is riskier. Because the value of capital, the aggregate $\mathbf{q}$, moves in lockstep with the investment-capital ratio $\mathbf{i}$, the steady-state Tobin's q is the highest in the economy with mandates and lowest in the laissez faire economy. The transition time to the steady state (conditional on remaining in state $\mathcal{G}$ at all time) in the market economy with the mandate is 10.9 years compared to 10.0 years for the planner's economy.

Now we quantify the society's willingness to pay (in units of consumption goods/dollars) for an optimal mandate. The optimal mandate generates a 52% welfare gain at the steady state where $\mathbf{n}^{ss} = 6.13\%$ and $b^{ss} = 0.0826$, which is almost identical to the first-best solution. This follows from a comparison with the laissez faire economy in which there is no decarbonization capital stock in equilibrium ($\mathbf{n}^{ss} = 0$) and the steady-state equilibrium value of $b^{ss} = 0.0542$. In sum, mitigation spending and macroeconomic variables in the market economy with mandates closely track the first-best.

6.3.2 Comparing Optimal Policies and Welfare Measure b

In Figure 1, we examine the optimal mitigation $\mathbf{x}$, investment $\mathbf{i}$, consumption $\mathbf{c}$, and a welfare measure (proportional to the certainty-equivalent wealth) b as functions of $\mathbf{n}$ in state $\mathcal{G}$. All these aggregates are functions of $\mathbf{n}$ in a given climate state $\mathcal{S}$. For all four panels, the blue solid lines depict the market economy solution under welfare-maximizing mandates and the red dashed lines describe the planner's first-best solution. We compare how closely the policy functions in the market economy with welfare-maximizing mandates track the social planner's first-best policies.

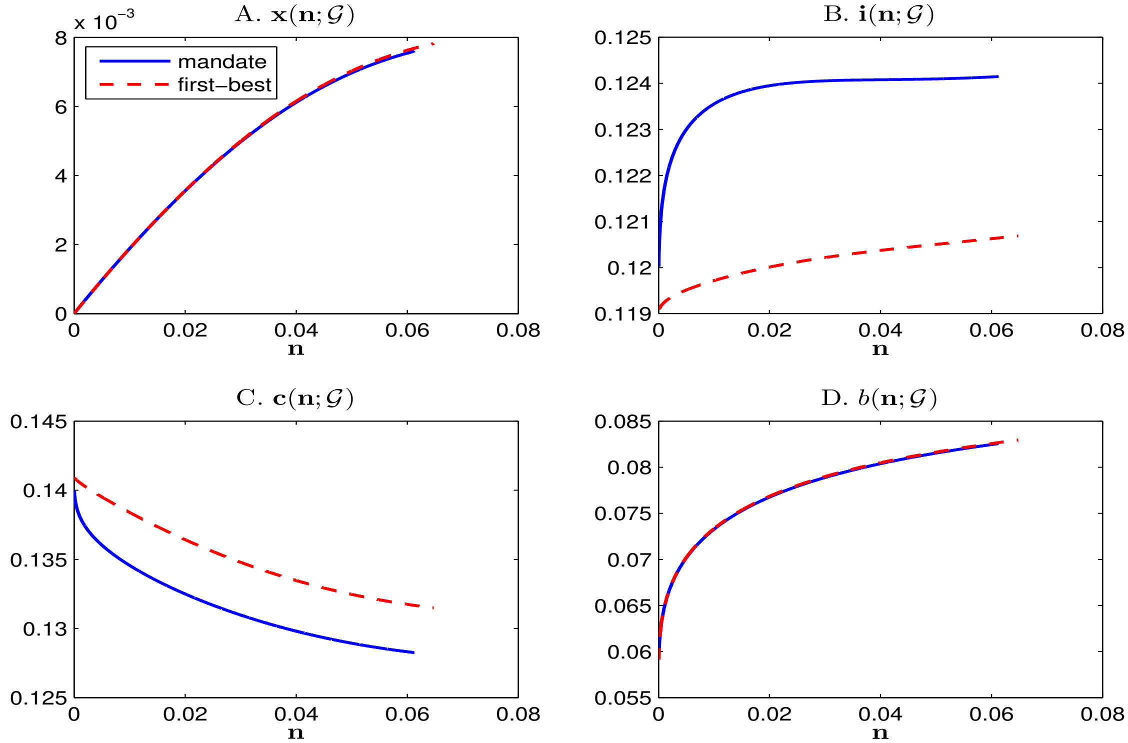

Figure 1: This figure plots aggregate mitigation spending $\mathbf{x}$, investment $\mathbf{i}$, consumption $\mathbf{c}$, and welfare measure b as functions of the scaled decarbonization capital stock $\mathbf{n}$ in state $\mathcal{G}$. The parameter values are reported in Table 1.

Panel A shows that the solution for the market economy with mandates closely tracks the planner's first-best all the way up to the steady-state: $\mathbf{n}^{ss} = 6.13\%$. It also shows that the first-best solution features a higher steady-state value: $\mathbf{n}^{FB} = 6.48\%$, which we discussed earlier. That is, even in the long run, the welfare-maximizing mandates still fall short of achieving the first-best.

Panel B shows that investment $\mathbf{i}$ is higher in the market economy with mandates than in the first-best economy in state $\mathcal{G}$. As we discussed earlier, firms in the market economy even with mandates do not fully take into account the impact of their capital accumulation decisions on the aggregate variables. At the margin, firms still over-invest relative to the first-best level. In contrast, the planner fully takes into account that more decarbonization capital stock $\mathbf{N}$ is necessary to effectively protect a larger economy (with a larger $\mathbf{K}$).

Since the resource constraint requires that the sum of $\mathbf{i}$, $\mathbf{c}$, and $\mathbf{x}$ equals the constant productivity A, consumption $\mathbf{c}$ is lower in the market economy with mandates than the first-best (panel C). This is because firms over-invest in the mandated market economy relative to the first-best and mitigation spending in the two economies are very close. Also as both $\mathbf{x}$ and $\mathbf{i}$ increase over time, scaled consumption $\mathbf{c}$ falls over time.

Panel D shows that the welfare measure $b(\mathbf{n};\mathcal{G})$ (proportional to certainty equivalent wealth) for the market economy with optimal mandates is almost identical to that in the first-best economy for all the levels of $\mathbf{n}$ up to the steady-state level of $\mathbf{n}^{ss} = 6.13\%$. This is good news as mandates are effectively incentivizing firms to contribute to decarbonization. However, the market economy with mandates still falls short of delivering the planner's first-best steady-state level of $\mathbf{n}^{FB} = 6.48\%$, which is about 5.4% higher than $\mathbf{n}^{ss} = 6.13\%$, as we discussed earlier.

6.3.3 Constrained Mitigation Spending: $\mathbf{x}_t \leq \overline{\mathbf{x}}$

Thus far, we have imposed no constraints on how much budget a firm can set aside for its mitigation spending. But in reality, often there are limits on how much a firm can contribute. Without loss of generality, we assume that the aggregate mitigation spending satisfies $\mathbf{x}_t \leq \overline{\mathbf{x}}$ at all t, where $\overline{\mathbf{x}}$ is the parameter measuring how tight this constraint is. This constraint applies to both the market economy with welfare-maximizing mandates and the planner's economy. For our quantitative analysis, we set $\overline{\mathbf{x}} = 0.35\%$. As the steady-state annual mitigation contribution is $\mathbf{x}^{ss} = 0.76\%$ in the market economy with optimal mandates, this constraint is reasonably tight as $\overline{\mathbf{x}} = 0.35\%$ is about 54% lower than the unconstrained steady-state annual mitigation spending $\mathbf{x}^{ss} = 0.76\%$ with optimal unconstrained mandates.

In Figure 2, we plot the optimal policy functions and welfare measure b for both the market economy with welfare-maximizing mandates and the planner's economy in state $\mathcal{G}$ with the aggregate mitigation spending satisfying the $\mathbf{x}_t \leq \overline{\mathbf{x}} = 0.35\%$ constraint. Panels A and D show that the market economy with welfare-maximizing mandates uses almost the same mitigation policy $\mathbf{x}$ and attains almost the same level of welfare as the planner facing

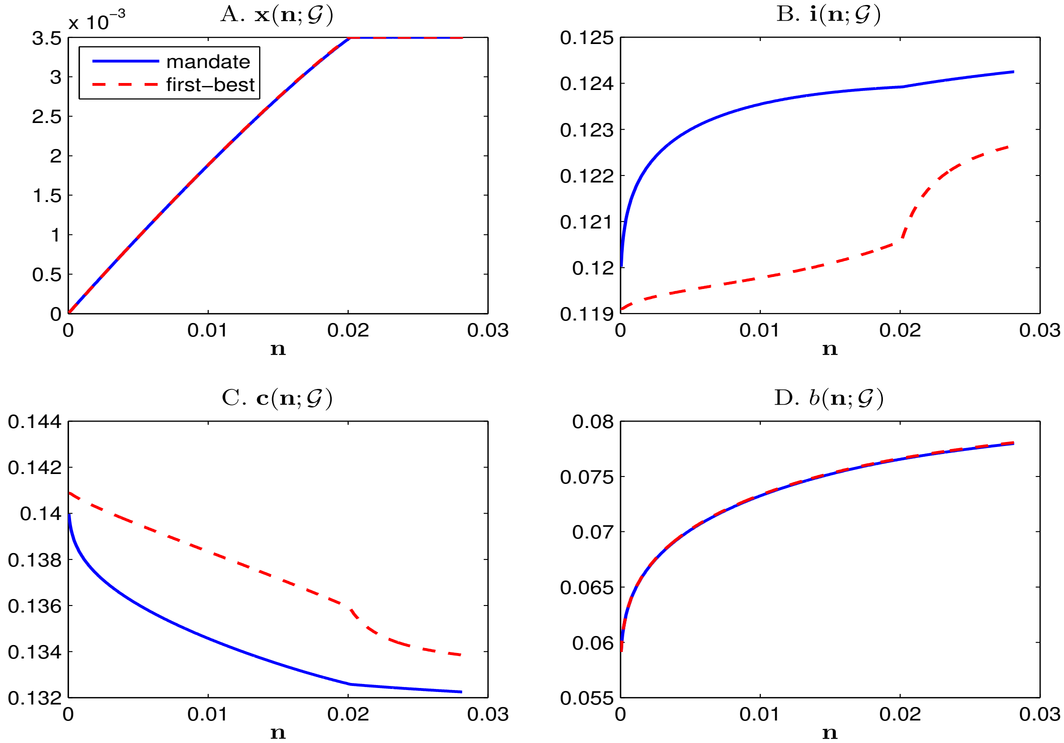

Figure 2: This figure plots the aggregate mitigation spending $\mathbf{x}$, investment $\mathbf{i}$, consumption $\mathbf{c}$, and welfare measure b in state $\mathcal{G}$ for both the planner's and mandated market economies with $\overline{\mathbf{x}} = 0.35\%$. All other parameter values are reported in Table 1.

the same $\mathbf{x}_t \le \overline{\mathbf{x}} = 0.35\%$ constraint. However, as for our baseline case without mitigation-spending constraints, the two economies generate different $\mathbf{i}$ and $\mathbf{x}$ dynamics. Panels B and C again confirm our earlier results that firms invest more in the market economy (even with mandates) than the planner's economy and hence households consume less in the mandated market economy than in the planner's economy.

6.3.4 Required m and Required Rate of Return Wedge for a Given α

In Figure 3, we plot the mandate $m(\mathbf{n};\mathcal{G})$ and the required rate of return wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ in panels A and B, respectively, for three levels of α (fraction of wealth pledged to the mandate): $0.1, 0.2, 0.3$.[31] The aggregate mitigation constraint $\mathbf{x} \le \overline{\mathbf{x}} = 0.35\%$ implies

[31] Gadzinski, Schuller and Vaccino (2018) estimate that the market value of global capital stock (including housing) in 2019 is close to \$800 trillion. Assuming Tobin's average q for global capital stock to be around 2, we infer that the stock of capital $\mathbf{K}$ is about \$400 trillion. To fund the Net-Zero pledges of \$100 trillion,

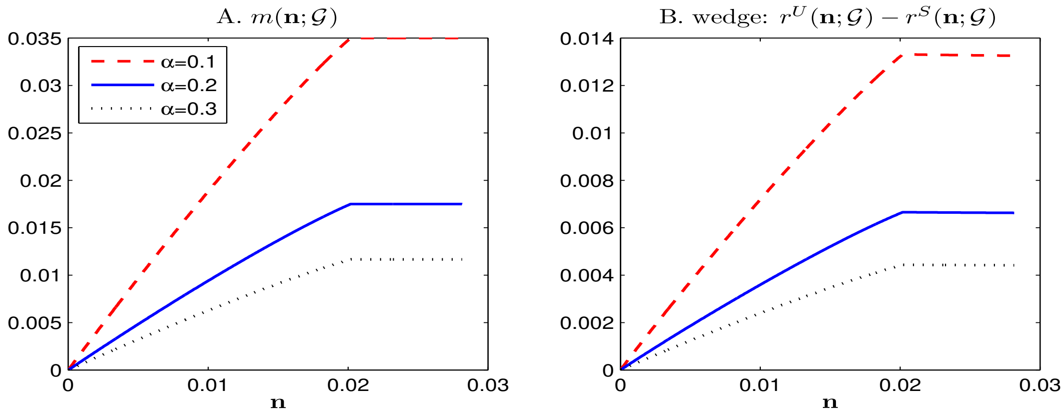

Figure 3: This figure plots firm-level mitigation spending mandate $m(\mathbf{n};\mathcal{G})$ and the required rate of return wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ for the $\alpha = 0.1, 0.2, 0.3$ cases in the market economy with welfare-maximizing mandates and the aggregate mitigation constraint: $\mathbf{x}_t \leq \overline{\mathbf{x}} = 0.35\%$. All other parameter values are reported in Table 1.

that a firm's scaled mitigation spending m must satisfy the constraint: $m \leq 0.35\%/\alpha$. For the $\alpha = 0.1$, $\alpha = 0.2$, and $\alpha = 0.3$ cases, the implied individual firm's constraints are $m \leq 3.5\%$, $m \leq 1.75\%$, and $m \leq 1.17\%$, respectively.

Panel A of Figure 3 shows that $m(\mathbf{n};\mathcal{G})$ increases with $\mathbf{n}$ but is capped for $\mathbf{n} \geq 0.02$ for all three cases. This is because $\mathbf{x}$ increases with $\mathbf{n}$ but is capped at $\overline{\mathbf{x}}$ for $\mathbf{n} \geq 0.02$. The higher the level of α, the less each firm has to contribute towards mitigation spending. For example, as we increase the total capital commitment to sustainable investment by 50% from $\alpha = 0.2$ to $\alpha = 0.3$, each firm's required contribution $m(\mathbf{n};\mathcal{G})$ decreases by one third. For example, when the aggregate mitigation constraint $\mathbf{x} \leq \overline{\mathbf{x}} = 0.35\%$ binds for $\mathbf{n} \geq 0.02$, each firm's mitigation spending decreases from 1.75% to 1.17% per annum.

Recall that the required rate of return wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ equals a firm's mitigation spending $m(\mathbf{n};\mathcal{G})$ divided by its average q, $\mathbf{q}(\mathbf{n};\mathcal{G})$. Because average q is much less sensitive to $\mathbf{n}$ than mitigation spending $m(\mathbf{n};\mathcal{G})$, the change of $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ mostly tracks the change of $m(\mathbf{n};\mathcal{G})$, which can be seen by comparing the two panels of Figure 3.

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the implied mandate requires 25% of aggregate wealth to be committed to sustainable firms.

6.3.5 Required α and Required Rate of Return Wedge for a Given m

We have characterized the welfare-maximizing mandate by (a) taking the total capital that can be committed towards sustainable investment (α) as given and (b) choosing the firm-level mandate m so that the economy can fund the aggregate level of mitigation spending $\mathbf{x}$. We could also derive the welfare-maximzing policy by (a) taking each firm's mitigation spending m as given and (b) solving for the required capital commitments in the aggregate economy α to fund the aggregate mitigation spending $\mathbf{x}$. Whether we solve for α taking m as given or we solve for m taking α as given yields the same welfare-maximizing aggregate mitigation spending $\mathbf{x}$, as long as the $\mathbf{x} \le \overline{\mathbf{x}}$ constraint is the same.

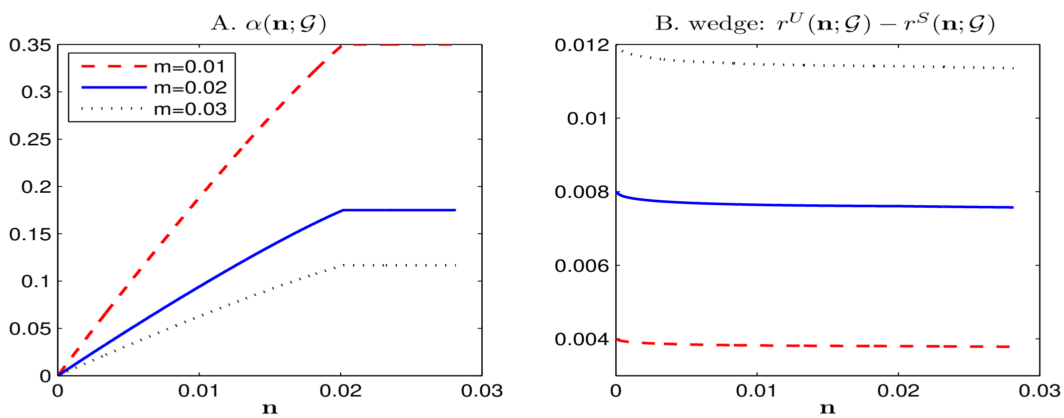

Figure 4: This figure plots required capital pledge for sustainable investment $\alpha(\mathbf{n};\mathcal{G})$ and the required rate of return wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ for the $m = 0.01, 0.02, 0.03$ cases in the market economy with welfare-maximizing mandates and the aggregate mitigation constraint: $\mathbf{x}_t \le \overline{\mathbf{x}} = 0.35\%$. The parameter values are reported in Table 1.

In Figure 4, we report the necessary capital pledge α for various levels of firm-level (scaled) mitigation spending m. In this figure, we continue to impose the same aggregate mitigation spending constraint $\mathbf{x} \le \overline{\mathbf{x}} = 0.35\%$ as in Subsubsection 6.3.4. Panel A shows that if each firm can spend 1% of its capital towards mitigation ($m = 0.01$), we then need 35% of the aggregate wealth to fund the maximal aggregate mitigation spending $\overline{\mathbf{x}} = 0.35\%$, i.e., $\alpha = 35\%$. The mitigation spending constraint $\mathbf{x}_t \le \overline{\mathbf{x}} = 0.35\%$ binds when $\mathbf{n} > 0.02$. If each firm can spend more towards mitigation ($m = 0.03$), the minimal level of required capital commitments (α) drops to around 12% at the steady state. Naturally, the required rate of return difference is lower when m is low (e.g., $m = 0.01$) since more capital (35% of total wealth) is committed to sustainable investing. But when m is high (e.g., $m = 0.03$), only about 11.7% of firms are committed to being sustainable in equilibrium and therefore these firms need a lower required rate of return in compensation, as we see in panel B. At the steady state, the required rate of return wedge is 1.14% per annum if the firm-level mandated (scaled) mitigation spending m is 1%, but drops significantly to only 0.38% per annum if the firm-level mandated mitigation spending m increases to 3%.

6.4 Optimal Transition Under a Welfare-Maximizing Mandate

In this subsection, we discuss the optimal transition under a welfare-maximizing mandate (the model analyzed in Subsection 3.4).

6.4.1 Decarbonization-to-productive Capital Ratio $\mathbf{n}_t$

In Figure 5, we plot the transition path of $\mathbf{n}_t$ over time t conditional on no climate transition from state $\mathcal{G}$ to $\mathcal{B}$ before reaching the steady state $\mathbf{n}^{ss}(\mathcal{G})$ in state $\mathcal{G}$. Due to adjustment costs, $\mathbf{n}_t$ gradually rises to the steady-state level $\mathbf{n}^{ss}$. We plot the transition paths for three different values of the adjustment cost parameter: $\eta_{\mathbf{N}} = 5$ (red dashed line), $\eta_{\mathbf{N}} = 5.5$ (solid blue line) and $\eta_{\mathbf{N}} = 5.85$ (black dotted line). We are interested in comparing the transition path in the market economy under optimal mandates with the planner's solution under a relatively pessimistic tipping point scenario (where the tipping point is expected to arrive in fifty years absent intervention, i.e., under the business-as-usual policy).

When $\eta_{\mathbf{N}} = 5$, the steady state of $\mathbf{n}_t$ is $\mathbf{n}^{ss} = 6.13\%$ in the mandated market economy and it takes about 11 years for $\mathbf{n}_t$ to reach $0.99 \times \mathbf{n}^{ss} = 6.07\%$, the 99% of the steady-state value. When we increase $\eta_{\mathbf{N}}$ from 5 to 5.5 the steady state decarbonization capital stock $\mathbf{N}$ decreases to 4.3% of the contemporaneous aggregate capital stock $\mathbf{K}$, i.e.,$\mathbf{n}^{ss} = 4.3\%$ and the transition time to 99% of the steady-tate value, $0.99 \times \mathbf{n}^{ss} = 4.26$, increases to 20 (almost doubling from 11 years). Finally, when we further increase $\eta_{\mathbf{N}}$ to 5.85, we see a dramatic

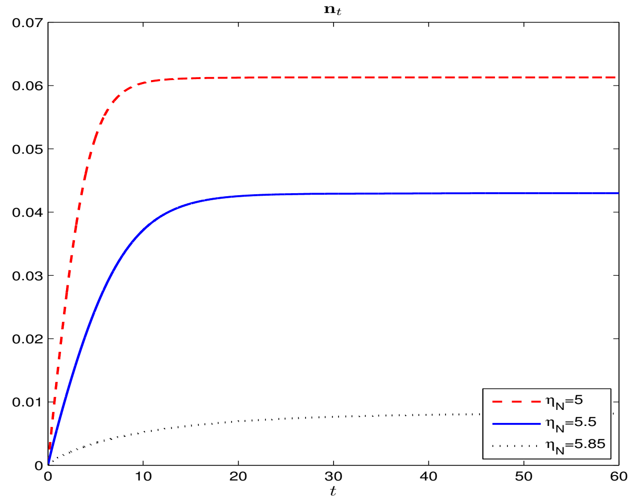

Figure 5: This figure plots the transition path of $\mathbf{n}_t$ in the market economy with optimal mandates conditional on being in state $\mathcal{G}$. The parameter values are reported in Table 1.

change in the transition path. The steady-state value of $\mathbf{n}$ drops to less than 1% and the transition time it takes to reach $0.99 \times \mathbf{n}^{ss}$ is around 50 years. In sum, the optimal transition path is highly sensitive to the decarbonization capital adjustment cost $\eta_{\mathbf{N}}$.

6.4.2 Mitigation, Investment, Consumption, and Welfare b_t under Mandates

In Figure 6, we examine the optimal mitigation $\mathbf{x}_t$, investment $\mathbf{i}_t$, consumption $\mathbf{c}_t$, and a welfare measure (proportional to the certainty-equivalent wealth) b_t transition dynamics conditional on being in state $\mathcal{G}$. In Panel A, $\mathbf{x}_t$ rises over time, reflecting the gradual buildup of decarbonization capital in the economy.[32] The higher is the adjustment cost of decarbonization capital relative to productive capital, the lower the level of mitigation spending. For $\eta_{\mathbf{N}} = 5$, the aggregate mitigation spending $\{\mathbf{x}_t\}$ reaches the steady-state value of $\mathbf{x}^{ss} = 0.76\%$. The steady-state annual contributions for the $\eta_{\mathbf{N}} = 5.5$ case equals $\mathbf{x}^{ss} = 0.59\%$, which is a 22% decrease from $\mathbf{x}^{ss} = 0.76\%$ for the baseline $\eta_{\mathbf{N}} = 5$ case. For the $\eta_{\mathbf{N}} = 5.85$ case, $\mathbf{x}^{ss} = 0.14\%$, which is 82% lower than $\mathbf{x}^{ss} = 0.76\%$ for the baseline $\eta_{\mathbf{N}} = 5$ case! In sum, the decarbonization capital adjustment cost is a critical parameter for

[32] While $\mathbf{x}_t$ increases over time, the mitigation spending/decarbonization capital stock ratio $\mathbf{x}_t/\mathbf{n}_t$ decreases over time conditional on being in state $\mathcal{G}$. This is because $\mathbf{n}_t$ is low in the early transition period.

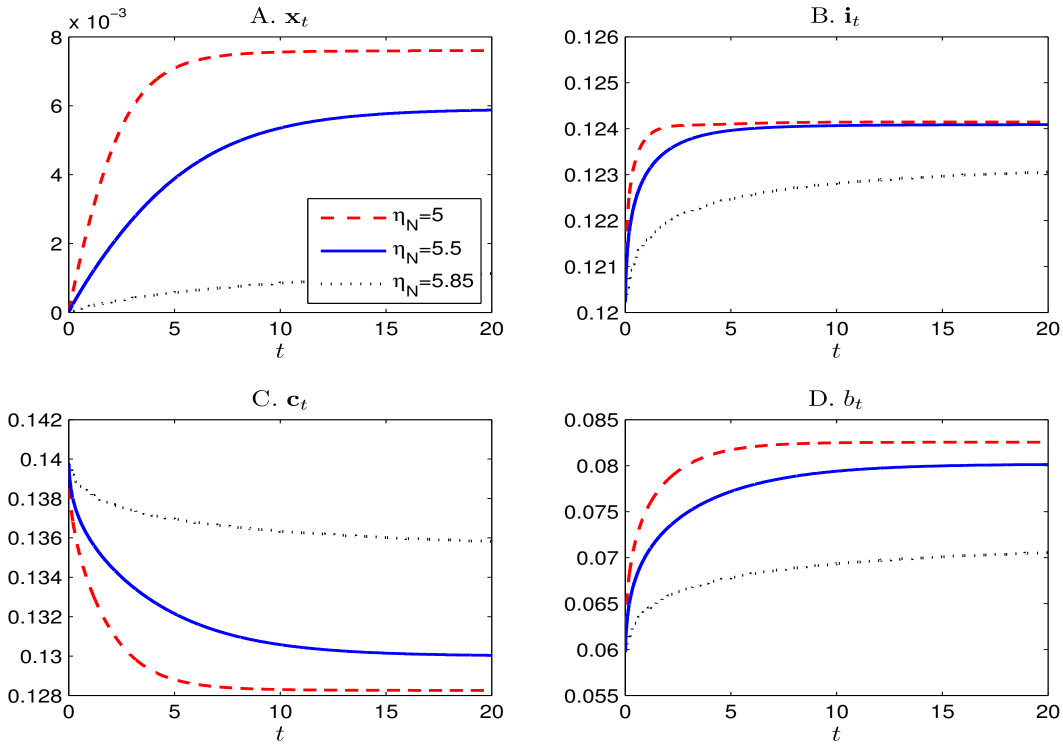

Figure 6: This figure plots the aggregate mitigation spending ($\mathbf{x}_t$), investment ($\mathbf{i}_t$), consumption ($\mathbf{c}_t$) and welfare measure (b_t) dynamics conditional on being in state $\mathcal{G}$. The parameter values are reported in Table 1.

our model.

In Panel B, $\mathbf{i}_t$ increases over time t as $\mathbf{n}_t$ increases over t. This is because the climate transition risk falls, which in turn makes the returns to investment rise. The lower is the adjustment cost of decarbonization capital, the higher the level of $\mathbf{i}_t$ since there is accumulation of decarbonization capital and less risk.

In Panel C, we see that consumption $\mathbf{c}$ falls over time as mitigation and investment ramp up due to the resource constraint as $\mathbf{c}_t = A - \mathbf{i}_t - \mathbf{x}_t$. Additionally, consumption rises as the adjustment cost increases since there is less spending on mitigation and also less investments.

In Panel D, we plot our measure of social welfare b_t, which is proportional to the agent's certainty equivalent wealth, over time conditional on being in state $\mathcal{G}$. Naturally, the higher is the adjustment cost of decarbonization capital, the lower the value of b. Moreover, the welfare measure b rises significantly over time as the economy decarbonizes. Focusing on the $\eta_N = 5$ case, we see that b_t rises from 0.06 at $t = 0$ to the steady-state value of 0.083. This 40% welfare gain is obviously very large.

Even at $\eta_N = 5.5$, we still obtain large welfare gains. However, for the $\eta_N = 5.85$ case, the welfare gain (again measured via the percentage change of b) is substantially lower as we transition from our current situation to the steady state. This is consistent with our earlier calculations in Figure 5 showing that the build-up of decarbonization capital is very sensitive to the adjustment cost of decarbonization capital relative to productive capital.

6.4.3 Mandated Spending for Qualifying Firms and Required Rate of Return Wedge

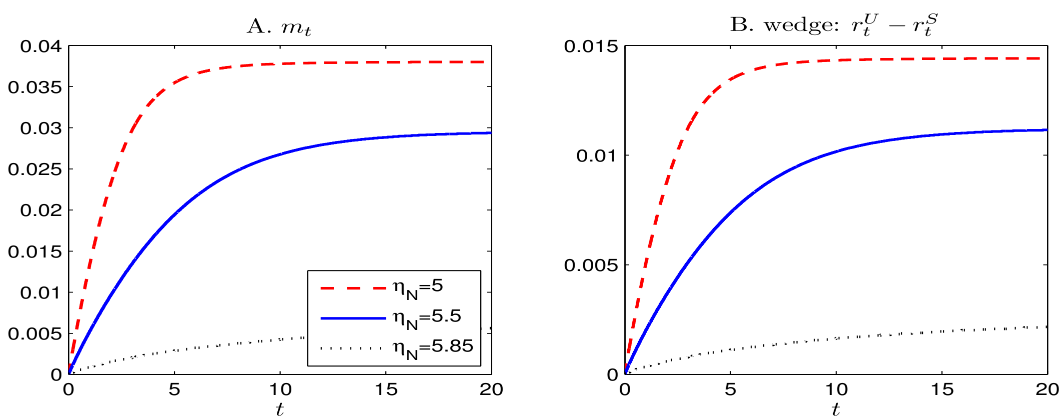

Figure 7: This figure plots the mitigation spending mandate (m_t) and the required rate of return wedge $r_t^U - r_t^S$ dynamics conditional on being in state $\mathcal{G}$. The parameter values are reported in Table 1.

In Figure 7, we present the optimal mandate m_t and the required rate of return wedge $r_t^U - r_t^S$ for the same three cases: $\eta_{\mathbf{N}} = 5, 5.5, 5.85$. Panels A and B show that both the qualifying standard for a firm (m_t) and the required rate of return wedge ($r_t^U - r_t^S$) increase with time t. Consider the $\eta_{\mathbf{N}} = 5$ case (dotted red lines). The mandate for a qualifying firm m_t peaks at the steady-state value of around 3.8% per annum.[33] That is, a firm would need to spend 3.8% of its capital stock per year on decarbonization to qualify for the

[33] This follows from $m^{ss} = \mathbf{x}^{ss}/\alpha = 0.76\%/20\% = 3.8\%$.

sustainable portfolio at the steady state. The sustainable firms are then compensated for their contributions with a significant required rate of return wedge $r_t^U - r_t^S = 1.4\%$ at the steady state in the market economy with mandates.[34]

Recall that as we increase the adjustment costs of decarbonization capital $\eta_{\mathbf{N}}$, both the steady-state $\mathbf{n}^{ss}$ and the required aggregate mitigation spending $\mathbf{x}$ decrease significantly. Therefore, the qualification standard for firms to be sustainable naturally falls and so do the required rate of return wedges. Note that the optimal ramp-up schedules of both m and required rate of return wedge $r^U - r^S$ are non-linear.

6.4.4 Tipping-point Arrival (ζ_t), Weather-disaster Arrival (λ_t), and Growth ($\mathbf{g}_t$)

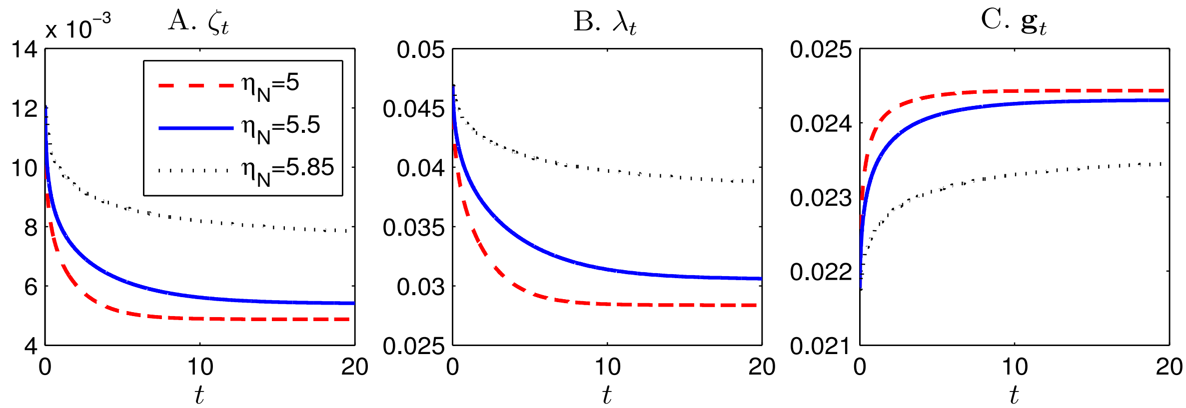

Figure 8: This figure plots the transition dynamics of tipping-point arrival rate (ζ_t), weather disaster arrival rate (λ_t), and the expected growth rate ($\mathbf{g}_t$) conditional on being in state $\mathcal{G}$. The parameter values are reported in Table 1.

We next highlight the mechanism for why social welfare is rising over the net-zero transition period. In panels A and B of Figure 8, we see that the tipping-point arrival rate and the disaster arrival rate λ_t falls over time t, as the society builds up the decarbonization capital. As the economy becomes more resilient, the expected growth rate $\mathbf{g}_t$ rises over time (panel C). There are three forces determining $\mathbf{g}_t$: the investment channel $\mathbf{i}$, the expected loss given a disaster arrival and the expected value destruction due to the expected tipping-point

[34]This follows from $r^U - r^S = \mathbf{x}^{ss}/(\alpha \mathbf{q}^{ss}) = 1.4\%$.

arrival, which can be seen from (63). Quantitatively, the investment channel $\phi(\mathbf{i}_t)$ dominates growth. Note that when the decarbonization capital adjustment costs $\eta_{\mathbf{N}}$ are high, gains from aggregate risk mitigation become much lower. This is because it is much more costly to mitigate risk and thus optimal for the society to reduce risk mitigation.

Even though the accumulation of decarbonization capital is entirely unproductive, economic growth can nonetheless rise in the net-zero transition due to the disaster-risk mitigation benefits of decarbonization. The logic behind this takeaway differs from the logic behind the projections from the European Union on the net-zero transition. The most recent climate briefing by European Union (2022) also sees positive growth projections from the net-zero transition. But their projection follows from its assumption that renewables will be highly efficient.

6.4.5 Asset Pricing and Valuation (Tobin's q)

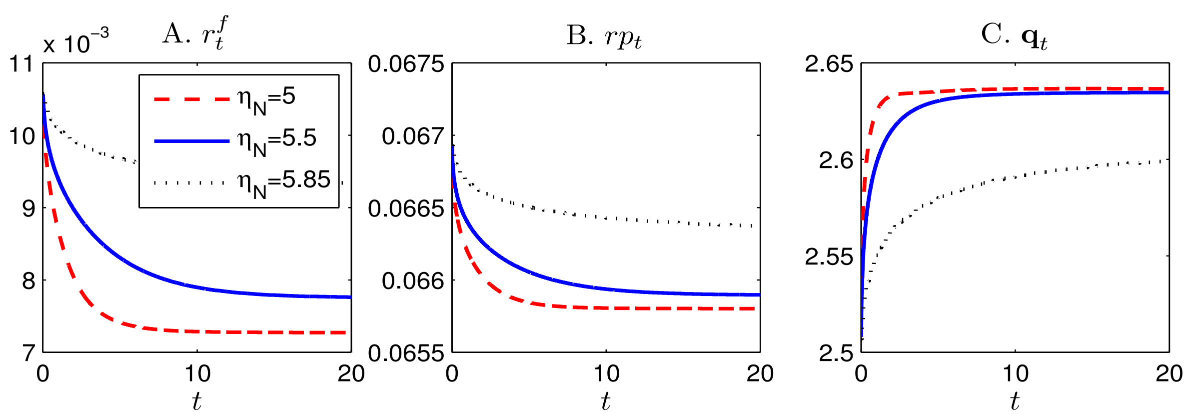

Figure 9: This figure plots the transition dynamics of the equilibrium interest rate (r_t^f), stock market risk premium (rp_t), and Tobin's average q for the aggregate capital stock ($\mathbf{q}_t$) conditional on being in state $\mathcal{G}$. The parameter values are reported in Table 1.

The benefit of decarbonizing the economy and reducing the damage of climate risks to physical capital stock is reflected in some of the asset prices. Panel B of Figure 9 shows that the market risk premium rp declines over time as the economy decarbonizes, while Panel C shows that Tobin's q modestly increases over time. However, the risk-free declines with

t in Panel A even though disaster risk is falling over the transition. In sum, asset prices (risk premium and Tobin's q) reflect the benefits of decarbonizing the economy and reducing climate disaster risks. But again, when adjustment costs of decarbonization is relatively higher, the impact on asset prices, government policies, and welfare are far weaker.

7 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate-disaster externalities. Firms that spend enough resources on mitigating these climate-disaster externalities qualify for sustainable finance mandates. These mandates incentivize otherwise *ex ante* identical unsustainable firms to become sustainable in order to lower their costs of capital. We present and solve a dynamic stochastic general-equilibrium model featuring the gradual accumulation of nonproductive but protective decarbonization capital to study the welfare consequences of sustainable finance.

Using our tractable model, we highlight some key takeaways by introducing the welfare-maximizing mandate into an otherwise laissez faire market economy. Despite being entirely unproductive, the disaster-risk mitigation benefits of decarbonization capital are such that investment, growth, and welfare are rising over time (and risk premia falling) as we approach the steady state. But the optimal transition path is highly sensitive to the relative adjustment costs of decarbonization to productive capital.

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Appendices

A Market Economy with Optimal Mandates

In this appendix, we provide additional technical details for the market economy with optimal Markovian mandates in Section 3.

First, we provide key intermediate steps for the household's problem.

A.1 Household's Optimization Problem

Using the household's wealth dynamics in state $\mathcal{S}$ given in (26), we obtain the following HJB equation for the household's value function $V(W, \mathbf{n}; \mathcal{S})$:

$$
\begin{aligned}
0 = \max_{C,\pi^S,H} & \left[r^f(\mathbf{n};\mathcal{S})W - C + \left(r^S(\mathbf{n};\mathcal{S})\pi^S + r^U(\mathbf{n};\mathcal{S})(1-\pi^S) - r^f(\mathbf{n};\mathcal{S}) \right) H + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))H \right] V_W \\
& + f(C,V;\mathcal{S}) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right]\mathbf{n}V_{\mathbf{n}} + \frac{\sigma^2 H^2 V_{WW}}{2} + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}HV_W \\
& + \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[V\left(W-(1-Z)H,\mathbf{n};\mathcal{S}\right) - V(W,\mathbf{n};\mathcal{S})\right] \\
& + \zeta(\mathbf{n};S)\left[V\left(W - \frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}H, \mathbf{n};\mathcal{S}'\right) - V(W,\mathbf{n};\mathcal{S})\right] . \qquad \text{(A.65)}
\end{aligned}
$$

subject to the investment mandate $\pi^S \geq \alpha$. In (A.65), we use the equilibrium property that the S- and the U-portfolio equilibrium returns have the same (diffusion and jump) risk exposures with probability one. Using (A.65), $W = H$, and $\pi = \alpha$, we obtain (27).[35]

The FOC for consumption C is the standard condition given by (28). The FOC for the portfolio allocation to the risky asset, H, is given by

$$
\begin{aligned}
0 = & \left[\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)r^U(\mathbf{n};\mathcal{S}) - r^f(\mathbf{n};\mathcal{S}) + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right] V_W \\
& + \sigma^2 HV_{WW} - \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[(1-Z)V_W\left(W-(1-Z)H,\mathbf{n};\mathcal{S}\right)\right] \\
& + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}V_W\left(W - \frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}H, \mathbf{n};\mathcal{S}'\right) . \qquad \text{(A.66)}
\end{aligned}
$$

Later, we use (A.66) to derive the equilibrium market return r^M.

Next, we derive equilibrium prices and allocations in the mandated market economy.

[35] Suppose that $r^S > r^U$ were true, the optimality condition for π^S would imply counterfactually $\pi^S \to \infty$, as (A.65) is linear in π^S. Since $\pi^S \to \infty$ cannot be an equilibrium, $r^S \leq r^U$ is necessary in equilibrium. Moreover, we can show that in equilibrium $r^S < r^U$ holds, which implies that the short-sale constraint $\pi^S \geq \alpha$ has to bind. This is because investors have incentives to short S firms otherwise. By combining the equilibrium condition $H = W$, we thus conclude that the household's value function satisfies the simplified HJB equation (27).

A.2 Market Equilibrium for a Given Mandate

First, a sustainable firm has no incentive to spend more on mitigation for its sustainability qualification than the minimal requirement m, which implies $x^S = \frac{X^S}{K^S} = m$. Second, in equilibrium, the representative household invests her entire wealth in the stock market and holds no risk-free asset: $H = W$ and $W = \mathbf{Q}^S + \mathbf{Q}^U$. Third, the representative agent's dollar-amount investment in the S portfolio equals the total market value of sustainable firms ($\pi^S = \alpha$) and her dollar-amount investment for the U portfolio equals the total market value of the U portfolio which includes all unsustainable firms ($\pi^U = 1 - \alpha$). Finally, goods market clears.

As in Pindyck and Wang (2013) and Hong, Wang, and Yang (2022), the risk-free asset holding is zero, $H = W = \mathbf{Q}^S + \mathbf{Q}^U = q^S(\mathbf{n};\mathcal{S})\mathbf{K}^S + q^U(\mathbf{n};\mathcal{S})\mathbf{K}^U = \mathbf{q}(\mathbf{n};\mathcal{S})(\mathbf{K}^S + \mathbf{K}^U) = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$, and $W^{\mathcal{J}} = ZW$. Additionally, using $\pi^S = \alpha$ and the portfolio allocation rule given in (A.66), we obtain

$$\begin{aligned} r^M(\mathbf{n};\mathcal{S}) &= r^f(\mathbf{n};\mathcal{S}) + \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right] \\ &\quad + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{-\gamma} - 1\right] \\ &= \alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)r^U(\mathbf{n};\mathcal{S})\,. \end{aligned} \tag{A.67}$$

As all firms have the same Tobin's q in equilibrium, using the investment FOCs (23) and (22) we conclude that both S and U firms invest at the same rate: $i^S(\mathbf{n};\mathcal{S}) = i^U(\mathbf{n};\mathcal{S}) = \mathbf{i}(\mathbf{n};\mathcal{S})$ and

$$\begin{aligned} \mathbf{q}(\mathbf{n};\mathcal{S}) &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{r^S(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})} \\ &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{r^U(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})}\,, \end{aligned} \tag{A.68}$$

where the expected growth ate is

$$\mathbf{g}(\mathbf{n};\mathcal{S}) = \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,. \tag{A.69}$$

Using $\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)r^U(\mathbf{n};\mathcal{S}) = r^M(\mathbf{n};\mathcal{S})$, $\mathbf{x} = \alpha m(\mathbf{n};\mathcal{S})$, and (A.68), we obtain

$$\begin{aligned} &\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{r^M(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})} \\ = &\frac{\alpha(A - \mathbf{i}(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S}))}{\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)r^U(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})} \\ &+\frac{(1-\alpha)(A - \mathbf{i}(\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S}))}{\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)r^U(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})} \\ = &\frac{\alpha\mathbf{q}(\mathbf{n};\mathcal{S})(r^S(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})) + (1-\alpha)\mathbf{q}(\mathbf{n};\mathcal{S})(r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S}))}{\alpha(r^S(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S})) + (1-\alpha)(r^U(\mathbf{n};\mathcal{S}) - \mathbf{g}(\mathbf{n};\mathcal{S}))} = \mathbf{q}(\mathbf{n};\mathcal{S})\,. \end{aligned} \tag{A.70}$$

The optimal consumption rule given in (30) implies

$$c(\mathbf{n};\mathcal{S}) = \frac{C}{\mathbf{K}} = \frac{C}{W}\mathbf{q}(\mathbf{n};\mathcal{S}) = \rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi}\mathbf{q}(\mathbf{n};\mathcal{S})\,. \tag{A.71}$$

And then substituting $c(\mathbf{n};\mathcal{S})$ given by (A.71) and the value function given in (29) into the HJB equation (27), we obtain

$$\begin{aligned}
0 \;=\;& \frac{1}{1-\psi^{-1}}\left(\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}-\rho\right)+\left(\alpha r^S(\mathbf{n};\mathcal{S})+(1-\alpha)r^U(\mathbf{n};\mathcal{S})-\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}+\lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))\right)\\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\\
&+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]\\
=\;& \frac{1}{1-\psi^{-1}}\left(\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}-\rho\right)+\left(r^M(\mathbf{n};\mathcal{S})-\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}+\lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))\right)\\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\\
&+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]. \qquad \text{(A.72)}
\end{aligned}$$

By using (A.70) and the goods market clear condition, we obtain

$$\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}=r^M(\mathbf{n};\mathcal{S})-\mathbf{g}(\mathbf{n};\mathcal{S})-\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}. \qquad \text{(A.73)}$$

Substituting (A.73) into (A.72) and using $c(\mathbf{n};\mathcal{S})=A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})$ and (A.71), we obtain

$$\begin{aligned}
&\frac{1}{1-\psi^{-1}}\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}-\rho\right)+\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\\
&+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{(A-\mathbf{i}(\mathbf{n};\mathcal{S}')-\mathbf{x}(\mathbf{n};\mathcal{S}'))\mathbf{q}(\mathbf{n};\mathcal{S})^{\psi}}{(A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S}))\mathbf{q}(\mathbf{n};\mathcal{S}')^{\psi}}\right)^{\frac{1-\gamma}{1-\psi}}-1\right]\\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}-\frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{S})+\mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{S})}{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}\right) \qquad \text{(A.74)}
\end{aligned}$$

which implies (37). Finally, we obtain the equilibrium risk-free rate formula (41) by substituting

$$r^M(\mathbf{n};\mathcal{S})=r^f(\mathbf{n};\mathcal{S})+\gamma\sigma^2+\lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right]+\zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{-\gamma}-1\right]$$

into (A.73). Next, we provide details on how to obtain the ODE (45) for $b(\mathbf{n};\mathcal{S})$, which equals the product of $u(\mathbf{n};\mathcal{S})$ and $\mathbf{q}(\mathbf{n};\mathcal{S})$. Then, we can obtain the welfare-maximizing mandate by choosing $\mathbf{x}$ to maximize $b(\mathbf{n};\mathcal{S})$.

A.3 Welfare-maximizing Markovian Mandate

Using (29) and $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ in equilibrium, we may rewrite the ODE (31) for $u(\mathbf{n};\mathcal{S})$ as:

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\psi^{-1}}\left[\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}-\rho\right]+\left(\alpha r^S(\mathbf{n};\mathcal{S})+(1-\alpha)r^U(\mathbf{n};\mathcal{S})+\lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))-\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)\\ & +\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\\ & +\zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{A.75}$$

Then using (44) and $\mathbf{q}(\mathbf{n};\mathcal{S})=\frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))}$, we obtain

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}}-\rho\right]+\left(\alpha r^S(\mathbf{n};\mathcal{S})+(1-\alpha)r^U(\mathbf{n};\mathcal{S})+\lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))\right)\\ & -\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\\ & +\zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{A.76}$$

Using (A.73) and (A.69) to simplify (A.76), we obtain:

$$\begin{aligned} 0 = & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))+\frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma}-1\right]\\ & +\left(\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right)\left(\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})}+\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right)-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \end{aligned} \tag{A.77}$$

Finally, using $b(\mathbf{n};\mathcal{S})=u(\mathbf{n};\mathcal{S})\times\mathbf{q}(\mathbf{n};\mathcal{S})$, we obtain (45).

B First Best

The following HJB equation for state $\mathcal{S}=\mathcal{G},\mathcal{B}$ characterize the planner's optimization problem:

$$\begin{aligned} 0 = \max_{\mathbf{C},\mathbf{i},\mathbf{x}} \; & f(\mathbf{C},J;\mathcal{S})+\phi(\mathbf{i})\mathbf{K}J_K+\omega(\mathbf{x}/\mathbf{n})\mathbf{N}J_{\mathbf{N}}+\frac{\mathbf{K}^2J_{\mathbf{KK}}+2\mathbf{NK}J_{\mathbf{KN}}+\mathbf{N}^2J_{\mathbf{NN}}}{2}\sigma^2\\ & +\lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[J\left(Z\mathbf{K},Z\mathbf{N};\mathcal{S}\right)-J(\mathbf{K},\mathbf{N};\mathcal{S})\right]+\zeta(\mathbf{n};\mathcal{S})\left[J(\mathbf{K},\mathbf{N};\mathcal{S}')-J(\mathbf{K},\mathbf{N};\mathcal{S})\right], \end{aligned} \tag{B.78}$$

subject to the aggregate resource constraint at all time t:

$$A\mathbf{K_t}=\mathbf{C_t}+\mathbf{i}_t\mathbf{K_t}+\mathbf{x}_t\mathbf{K_t}\,. \tag{B.79}$$

The FOC for the scaled investment $\mathbf{i}$ is

$$f_{\mathbf{C}}(\mathbf{C},J;\mathcal{S})=\phi'(\mathbf{i})J_{\mathbf{K}}(\mathbf{K},\mathbf{N};\mathcal{S})\,. \tag{B.80}$$

The FOC for the scaled aggregate mitigation spending $\mathbf{x}$ is

$$f_{\mathbf{C}}(\mathbf{C}, J; \mathcal{S}) = \omega'(\mathbf{x}/\mathbf{n}) J_{\mathbf{N}}(\mathbf{K}, \mathbf{N}; \mathcal{S}), \tag{B.81}$$

for the economically interesting case where the first-best mitigation spending is strictly positive: $\mathbf{x} > 0$.[36] The FOCs (B.80) and (B.81) imply the following condition:

$$\frac{\omega'(\mathbf{x}/\mathbf{n})}{\phi'(\mathbf{i})} = \frac{J_{\mathbf{K}}(\mathbf{K}, \mathbf{N}; \mathcal{S})}{J_{\mathbf{N}}(\mathbf{K}, \mathbf{N}; \mathcal{S})}. \tag{B.82}$$

The left side of (B.82) is the ratio between the marginal investment efficiency for $\mathbf{N}$, $\omega'(\mathbf{x}/\mathbf{n})$, and the marginal investment efficiency for $\mathbf{K}$, $\phi'(\mathbf{i})$. The right side of (B.82) is the ratio between the marginal (utility) value of $\mathbf{N}$ and the marginal (utility) value of $\mathbf{K}$.

Substituting the agent's value function (42) into the FOCs (B.80)-(B.81) and the HJB equation (B.78) and simplifying these equations, we obtain (50), (51), and (52) for state $\mathcal{S} = \mathcal{G}, \mathcal{B}$.

C Market Economy with Mandates versus First Best

In this appendix, we first show why the optimally mandated market economy does not generate the first-best outcome (Subsection C.1) and then provide details on how to attain the first-best by introducing optimal investment taxes into the mandated market economy (Subsection 5.2).

C.1 Differences between the Optimally Mandated Market Economy and First Best

First we summarize the key equations for the optimally mandated market and first-best economies.

C.1.1 First Best

The planner chooses $\mathbf{i}$ and $\mathbf{x}$ to maximize the welfare measure $b(\mathbf{n}; \mathcal{S})$ given by the following ODE:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}} \left[\left(\frac{A - \mathbf{i}(\mathbf{n}; \mathcal{S}) - \mathbf{x}(\mathbf{n}; \mathcal{S})}{b(\mathbf{n}; \mathcal{S})} \right)^{1-\psi^{-1}} - 1 \right] + \phi(\mathbf{i}(\mathbf{n}; \mathcal{S})) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n}; \mathcal{S})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] \\ & + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{S})) \right] \frac{\mathbf{n} b'(\mathbf{n}; \mathcal{S})}{b(\mathbf{n}; \mathcal{S})} + \frac{\zeta(\mathbf{n}; \mathcal{S})}{1-\gamma} \left[\left(\frac{b(\mathbf{n}; \mathcal{S}')}{b(\mathbf{n}; \mathcal{S})} \right)^{1-\gamma} - 1 \right], \end{aligned} \tag{C.83}$$

which implies the following FOC for investment:

$$\rho \left(\frac{A - \mathbf{i}(\mathbf{n}; \mathcal{S}) - \mathbf{x}(\mathbf{n}; \mathcal{S})}{b(\mathbf{n}; \mathcal{S})} \right)^{-\psi^{-1}} = \phi'(\mathbf{i}(\mathbf{n}; \mathcal{S})) b(\mathbf{n}; \mathcal{S}) - \phi'(\mathbf{i}(\mathbf{n}; \mathcal{S})) \mathbf{n} b'(\mathbf{n}; \mathcal{S}), \tag{C.84}$$

and the FOC for mitigation spending $\mathbf{x}$ given in (51).

[36] Otherwise, $\mathbf{x} = 0$ as mitigation cannot be negative.

C.1.2 Mandated Market Economy

In contrast, in the mandated market economy, an individual firm chooses i to maximize its market value, i.e., $q(\mathbf{n};\mathcal{S})$, taking the aggregate mitigation spending $\mathbf{x}$ and the evolution of the scaled decarbonization capital stock $\mathbf{n}$ as well as asset prices as given. Then, in equilibrium, an individual firm's investment-capital ratio i equals $\mathbf{i}$, the aggregate investment-capital ratio in the economy.

Substituting the equilibrium results $q^S(\mathbf{n};\mathcal{S}) = q^U(\mathbf{n};\mathcal{S}) = \mathbf{q}(\mathbf{n};\mathcal{S})$ and $i^S(\mathbf{n};\mathcal{S}) = i^U(\mathbf{n};\mathcal{S}) = \mathbf{i}(\mathbf{n};\mathcal{S})$ into (22), we obtain the following equation for the aggregate Tobin's q, $\mathbf{q}(\mathbf{n};\mathcal{S})$:

$$
\begin{aligned}
r^j(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S}) = & \max_i \; cf^j(\mathbf{n};\mathcal{S}) + (\phi(i) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)))\mathbf{q}(\mathbf{n};\mathcal{S}) \\
& + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S}) + \zeta(\mathbf{n};\mathcal{S})(\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S}))\,.
\end{aligned} \tag{C.85}
$$

Since the preceding equation applies to both S and U firms, we may multiply α and $1-\alpha$ on both sides of the preceding equation for type-S and type-U firms, respectively. Doing so yields two equations. Summing up these two equations yields an equation for $\mathbf{q}(\mathbf{n};\mathcal{S})$. Dividing the two sides of this new equation and rearranging terms, we obtain:

$$
\begin{aligned}
0 = & \max_i \; \frac{A - i - \mathbf{x}}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \phi(i) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))) - r^M(\mathbf{n};\mathcal{S}) \\
& + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,.
\end{aligned} \tag{C.86}
$$

Next, substituting (29) into (27) and using the equilibrium condition $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$, we obtain the following equation for $u(\mathbf{n};\mathcal{S})$:

$$
\begin{aligned}
0 \;=\; & \max_c \; \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{c}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + r^M(\mathbf{n};\mathcal{S}) + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \frac{c}{\mathbf{q}(\mathbf{n};\mathcal{S})} \\
& + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
& + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right],
\end{aligned} \tag{C.87}
$$

Substituting the resource constraints $c = A - i - \mathbf{x}$ into (C.87), we obtain:

$$
\begin{aligned}
0 \;=\; & \max_i \; \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A - i - \mathbf{x}}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + r^M(\mathbf{n};\mathcal{S}) + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \frac{A - i - \mathbf{x}}{\mathbf{q}(\mathbf{n};\mathcal{S})} \\
& + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
& + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S}) - \mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right].
\end{aligned} \tag{C.88}
$$

Summing up (C.86) and (C.88), we obtain the following:

$$
\begin{aligned}
0 = \max_i \; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-i-\mathbf{x}}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \phi(i) + (\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S})))\left(\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} + \frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right) \\
& - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \qquad (C.89)
\end{aligned}
$$

Now using $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times \mathbf{q}(\mathbf{n};\mathcal{S})$, we obtain

$$
\begin{aligned}
0 = \max_i \; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-i-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \phi(i) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \\
& - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{S}')}{b(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \qquad (C.90)
\end{aligned}
$$

The firm's investment FOC for i implied by (C.90) is:

$$
\rho\left(\frac{A-i-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \phi'(i)b(\mathbf{n};\mathcal{S}). \qquad (C.91)
$$

Since in equilibrium firm-level's investment i equals the aggregate $\mathbf{i}$. Therefore, the following equation characterizes $\mathbf{i}$:

$$
\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S}). \qquad (C.92)
$$

In equilibrium, the welfare measure $b(\mathbf{n};\mathcal{S})$ then satisfies:

$$
\begin{aligned}
0 = \; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \phi(\mathbf{i}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} \\
& - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{S}')}{b(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \qquad (C.93)
\end{aligned}
$$

While the two ODEs, (C.83) and (C.93), at the aggregate level for the mandated market and first-best economies are the same, the two equations for $\mathbf{i}$, (C.84) and (C.92), are different.[37] Therefore, the resource allocations in the two economies are different.

Importantly, in a market economy regardless of mandates, a firm takes the evolution of the scaled aggregate decarbonization capital $\mathbf{n}$ as given. In contrast, in the first-best economy, when choosing investment $\mathbf{i}$ the planner internalizes the impact of aggregate $\mathbf{i}$ on the $\mathbf{n}$ process. The aggregate investment $\mathbf{i}$ in the optimally mandated market economy differs from that in the first-best economy because firms does not internalize the benefit of aggregate risk mitigation.

Next, we prove that by introducing an optimally chosen tax that depends on the difference between a firm's investment-capital ratio i and the aggregate $\mathbf{i}$ into the market economy with optimal mandates restores the first-best.

[37] Note that the FOCs (functional forms) for mitigation spending $\mathbf{x}$ in the mandated market economy and the first-best economy are the same (that is, (46) and (51) are the same.)

C.2 Introducing Investment Taxes into the Mandated Market Economy Restores First Best

Consider introducing the following optimal tax given in (55) as

$$\widehat{\tau}^j(\mathbf{n};\mathcal{S}) = \left[\phi(i^j) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right] \mathbf{q}(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\,, \tag{C.94}$$

into the market economy with mandates.

The following HJB equation characterizes the firm's value function in climate state $\mathcal{S}$:

$$\begin{aligned} r^j(\mathbf{n};\mathcal{S})Q^j(K^j,\mathbf{n};\mathcal{S}) = \max_{I^j}\ & CF^j(\mathbf{n};\mathcal{S}) - \widehat{\tau}^j(\mathbf{n};\mathcal{S})K^j + \Phi(I^j,K^j)Q^j_K(K^j,\mathbf{n};\mathcal{S}) \\ & + \frac{1}{2}(\sigma K^j)^2 Q^j_{KK}(K^j,\mathbf{n};\mathcal{S}) + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}Q^j_{\mathbf{n}}(K^j,\mathbf{n};\mathcal{S}) \\ & + \lambda(\mathbf{n};\mathcal{S})\mathbb{E}\left[Q^j(ZK^j,\mathbf{n};\mathcal{S}) - Q^j(K^j,\mathbf{n};\mathcal{S})\right] \\ & + \zeta(\mathbf{n};\mathcal{S})(Q^j(K^j,\mathbf{n};\mathcal{S}') - Q^j(K^j,\mathbf{n};\mathcal{S}))\,. \end{aligned} \tag{C.95}$$

Using the homogeneity property of our model, we obtain the following ODE for $q^j(\mathbf{n};\mathcal{S})$:

$$r^j(\mathbf{n};\mathcal{S})q^j(\mathbf{n};\mathcal{S}) = \max_{i^j}\ cf^j(\mathbf{n};\mathcal{S}) + (\phi(i^j) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)))q^j(\mathbf{n};\mathcal{S}) \tag{C.96}$$

$$\begin{aligned} & - \left[\phi(i^j) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right] q(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}q^j_{\mathbf{n}}(\mathbf{n};\mathcal{S}) \\ & + \zeta(\mathbf{n};\mathcal{S})(q^j(\mathbf{n};\mathcal{S}') - q^j(\mathbf{n};\mathcal{S}))\,. \end{aligned} \tag{C.97}$$

The FOC for investment i^j is given by

$$1 = \phi'(i^j)\left(q^j - q(\mathbf{n};\mathcal{S})\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)\,. \tag{C.98}$$

Substituting $i^U = i^S = \mathbf{i}$ and $q^U = q^S = \mathbf{q}$ into (C.96), we obtain the following equilibrium pricing equation for $\mathbf{q}$:

$$\begin{aligned} r^j(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S}) = & \ cf^j(\mathbf{n};\mathcal{S}) + (\phi(\mathbf{i}(\mathbf{n};\mathcal{S}))) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)))\mathbf{q}(\mathbf{n};\mathcal{S}) \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S}) + \zeta(\mathbf{n};\mathcal{S})(\mathbf{q}(\mathbf{n};\mathcal{S}') - \mathbf{q}(\mathbf{n};\mathcal{S}))\,, \end{aligned} \tag{C.99}$$

which implies (A.68) and (A.70) are still held.

Since $i^U = i^S = \mathbf{i}$ and $q^U = q^S = \mathbf{q}$ in equilibrium, the following equilibrium condition between the aggregate investment-capital ratio ($\mathbf{i}$) and Tobin's q for the aggregate capital stock ($\mathbf{q}$) holds:

$$1 = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\mathbf{q}(\mathbf{n};\mathcal{S})\left(1 - \frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)\,, \tag{C.100}$$

which implies

$$\frac{b(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\left(b(\mathbf{n};\mathcal{S}) - \mathbf{n}b'(\mathbf{n};\mathcal{S})\right)\,. \tag{C.101}$$

Rewriting the optimal consumption rule (30) and using the equilibrium restrictions, we obtain

$$c(\mathbf{n};\mathcal{S}) = \frac{C}{\mathbf{K}} = \frac{C}{W}\mathbf{q}(\mathbf{n};\mathcal{S}) = \rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi}\mathbf{q}(\mathbf{n};\mathcal{S}) = \rho^{\psi}u(\mathbf{n};\mathcal{S})^{-\psi}b(\mathbf{n};\mathcal{S})\,, \tag{C.102}$$

which implies

$$\frac{b(\mathbf{n};\mathcal{S})^{\psi}}{\mathbf{q}(\mathbf{n};\mathcal{S})^{\psi}} = \rho^{\psi}\frac{b(\mathbf{n};\mathcal{S})}{c(\mathbf{n};\mathcal{S})} = \rho^{\psi}\frac{b(\mathbf{n};\mathcal{S})}{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}\,. \tag{C.103}$$

And then combining (C.101) and (C.103), we obtain the FOC in equilibrium at the aggregate level for $\mathbf{i}$ is then given by (58), which is the same at the optimal investment under FB as given in (C.84).

Next, we verify that the ODE for $b(\mathbf{n};\mathcal{S})$ in the mandated market economy with investment taxes is the same as the ODE (52) for $b(\mathbf{n};\mathcal{S})$ in the first-best economy.

Recall that in the representative agent's optimization problem, we have the following ODE for $u(\mathbf{n};\mathcal{S})$:

$$\begin{aligned} 0 \;=\; & \frac{\rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi}-\rho}{1-\psi^{-1}} + \alpha r^{S}(\mathbf{n};\mathcal{S}) + (1-\alpha)r^{U}(\mathbf{n};\mathcal{S}) - \rho^{\psi}u(\mathbf{n};\mathcal{S})^{1-\psi} + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ & + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \end{aligned} \tag{C.104}$$

Using (30) and the equilibrium result $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$, we may rewrite the ODE (C.104) as:

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\psi^{-1}}\left[\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} - \rho\right] + \alpha r^{S}(\mathbf{n};\mathcal{S}) + (1-\alpha)r^{U}(\mathbf{n};\mathcal{S}) + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} \\ & + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ & + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \end{aligned} \tag{C.105}$$

Then using (58) and $\mathbf{q}(\mathbf{n};\mathcal{S}) = \frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\left(1-\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)}$, we obtain

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \left(\alpha r^{S}(\mathbf{n};\mathcal{S}) + (1-\alpha)r^{U}(\mathbf{n};\mathcal{S}) + \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z))\right) \\ & - \frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ & + \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})} + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \end{aligned} \tag{C.106}$$

Using (A.70) and $\mathbf{g}(\mathbf{n};\mathcal{S}) = \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \lambda(\mathbf{n};\mathcal{S})(1-\mathbb{E}(Z)) - \zeta(\mathbf{n};\mathcal{S})\frac{\mathbf{q}(\mathbf{n};\mathcal{S})-\mathbf{q}(\mathbf{n};\mathcal{S}')}{\mathbf{q}(\mathbf{n};\mathcal{S})}$ to simplify (C.106), we obtain:

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] + (\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S})))\left(\frac{\mathbf{n}u'(\mathbf{n};\mathcal{S})}{u(\mathbf{n};\mathcal{S})} + \frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\right) + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{S}')\mathbf{q}(\mathbf{n};\mathcal{S}')}{u(\mathbf{n};\mathcal{S})\mathbf{q}(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \tag{C.107}$$

Finally, using $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times \mathbf{q}(\mathbf{n};\mathcal{S})$, we can simplify (C.107) to the following ODE for $b(\mathbf{n};\mathcal{S})$:

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{1-\psi^{-1}} - 1\right] + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})} + \phi(\mathbf{i}(\mathbf{n};\mathcal{S})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] + \frac{\zeta(\mathbf{n};\mathcal{S})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{S}')}{b(\mathbf{n};\mathcal{S})}\right)^{1-\gamma} - 1\right]. \tag{C.108}$$

This ODE is the same as the ODE for $b(\mathbf{n};\mathcal{S})$ given in (52) for the first-best economy.

In sum, we have shown that by introducing the investment tax (55) into the mandated market economy allows us to attain the first-best outcomes.